Improvement quantum teleportation via the pair coherent states

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Improvement quantum teleportation via the pair coherent states

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Abstract. In this paper, we present two methods to improve the average fidelity in the quantum teleportation processes to teleport a coherent state via the pair coherent states. We use the measuring of the orthogonal quadrature components in the first protocol and the photon number sum and phase difference in the second. The results show that the average fidelity in the first protocol is enhanced by performing a photon number shifting at a receiver, and F_{av} can approach to the unit if q becomes very big. In the second protocol, the average fidelity increases with increasing the amplitude of the pair coherent states. It approaches to the unit in the limit of the big amplitude values of the pair coherent states and small of the input state. The average fidelity of both methods is greatly improved comparing with the Gabris-Agarwal protocol and it approaches to unit depending on the parameters involved.

1. Introduction

The quantum teleportation is a disembodied information transfer process. After ideal Bennett's protocol [1], the quantum teleportation has developed further to many aspects. For example, the protocols were proposed by using the measuring of the orthogonal quadrature components [2], [3], the Bell measurement [4], the photon number difference and phase sum [5], the photon number sum and phase difference [6], and the multi-particle entanglement sources [7], [8].

According to the quantum teleportation processes, the nonclassical states play a very important role as the entanglement sources to teleport the arbitrary states [9], [10], [11], [12], [13], [14], [15]. One of the nonclassical states that we are interested in studying is the pair coherent state [16]. Recently, some states by generalization of the pair coherent states have been introduced and their nonclassical states have been studied [17], [18], [19]. The pair coherent states were defined by Agarwal as the eigenstates of the pair boson annihilation operators $\hat{a}\hat{b}$ and photon number operators difference $\hat{N}_b - \hat{N}_a = \hat{a}^+\hat{a} - \hat{b}^+\hat{b}$ in the form:

$$\begin{aligned} \hat{a}\hat{b}|\Psi_q\rangle_{ab} &= \xi|\Psi_q\rangle_{ab}, \\ (\hat{N}_b - \hat{N}_a)|\Psi_q\rangle_{ab} &= q|\Psi_q\rangle_{ab}, \end{aligned} \tag{1}$$

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where ξ is complex, $\xi = |\xi|e^{i\varphi}$, $0 \leq \varphi \leq 2\pi$, q is positive integer. In Fock states, such states read

$$|\Psi_q\rangle_{ab} = \mathcal{N}_q(|\xi|) \sum_{n=0}^{\infty} \frac{\xi^n}{\sqrt{n!(n+q)!}} |n, n+q\rangle_{ab}, \tag{2}$$

where

$$\mathcal{N}_q(|\xi|) = \frac{1}{\sqrt{\xi^q I_q(2|\xi|)}}, \tag{3}$$

is the normalized factor, and $I_q(2|\xi|)$ is the modified Bessel function.

The quantum teleportation process by using Gabris-Agarwal protocol [11], in which the pair coherent states as the entanglement sources are feasible but the maximum of the average fidelity in the quantum teleportation process is only 0.75884. In this paper, we improve the quantum teleportation via the pair coherent states by using two different ways as the orthogonal quadrature components [3] and the photon number sum and phase difference protocols [6]. We show that in both ways, the average fidelity of the quantum teleportation process can be greatly improved and it approaches to the unit corresponding on the maximum fidelity depending on the appropriate parameters.

2. The quantum teleportation uses the orthogonal quadrature components protocol

The orthogonal quadrature components protocol was introduced by Furusawa *et al* [2] and developed more further by several authors [3], [5], [11], [14]. Using this protocol, we assume that A and B parties are shared by the pair coherent states that are given in Eqs. (2) and (3). The A party has mode a, and mode b is given for the B party. A wants to teleport to B a coherent state $|\alpha\rangle_c$ of mode c. To begin the process, A performs a combination of the three-mode state is

$$|\Phi_{in}\rangle_{abc} = |\Psi_q\rangle_{ab}|\alpha\rangle_c. \tag{4}$$

Next, he measures the orthogonal quadrature components [3] on modes a and c. After measurement, a non-normalized state of B becomes

$$|\Phi\rangle_b = \frac{e^{(\beta^*\alpha - \beta\alpha^*)/2} e^{-|\alpha-\beta|^2/2}}{\sqrt{\pi}} \sum_{n=0}^{\infty} \frac{c_{n,q}(\alpha-\beta)^n}{\sqrt{n!}} |n+q\rangle_b, \tag{5}$$

where

$$c_{n,q} = \mathcal{N}_q(|\xi|) \frac{\xi^n}{\sqrt{n!(n+q)!}}. \tag{6}$$

Then, A sends the measurement results to B by a classical channel. When B receives these data, first, he performs a photon number shifting by using operator [20]

$$\hat{U}_q = \sum_{j=0}^{\infty} |j\rangle\langle j+q|, \tag{7}$$

then, uses the displacement operator $\hat{D}(\beta)$, and the normalized output state reads

$$|\Phi_{nor.}\rangle_{out} = \frac{e^{(\beta^*\alpha - \beta\alpha^*)/2} e^{-|\alpha-\beta|^2/2}}{\sqrt{P(\beta)}\sqrt{\pi}} \sum_{n=0}^{\infty} \frac{c_{n,q}(\alpha-\beta)^n}{\sqrt{n!}} \hat{D}(\beta)|n\rangle, \tag{8}$$

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where the probability of the measurement $P(\beta) = {}_b\langle\Phi|\Phi\rangle_b$ is in the form

$$P(\beta) = \frac{e^{-|\alpha-\beta|^2}}{\pi}\sum_{n=0}^{\infty}\frac{|c_{n,q}|^2|\alpha-\beta|^{2n}}{n!}. \tag{9}$$

In case $\xi = |\xi|$, the average fidelity of the quantum teleportation process is

$$\begin{aligned} F_{av} &= \int |\langle\alpha|\Phi\rangle_{out}|^2 d^2\beta \\ &= \frac{\mathcal{N}_q^2(|\xi|)}{2}\sum_{m,n=0}^{\infty}\frac{|\xi/2|^{m+n}(m+n)!}{m!n!\sqrt{m!n!(m+q)!(n+q)!}}. \end{aligned} \tag{10}$$

3. The quantum teleportation uses the measuring of photon number sum and phase difference protocol

In the measuring of the photon number sum and phase difference protocol [6], the input state $|\alpha\rangle_c$ is expanded in Fock space as

$$|\alpha\rangle_c = \sum_{m=0}^{\infty} d_m|m\rangle_c, \tag{11}$$

where $|m\rangle_c$ is the Fock state, and

$$d_m = \frac{e^{-|\alpha|^2/2}\alpha^m}{\sqrt{m!}}. \tag{12}$$

We rewrite Eq. (4) as

$$|\Phi_{in}\rangle_{abc} = \sum_{m,n=0}^{\infty} c_{n,q}d_m|n, n+q, m\rangle_{abc}. \tag{13}$$

A measures the photon number sum and phase difference on two modes a and c, a state of the system becomes

$$|\Phi_b\rangle = \frac{{}_{ac}\langle\phi_N^-|\Phi_{in}\rangle_{abc}}{\sqrt{P}}, \tag{14}$$

where $P = \langle\Psi_b|\Psi_b\rangle$ is probability to obtain the photon numbers sum N and phase difference ϕ^-, and $|\phi_N^-\rangle_{ac}$ is the eigenstate of the photon number sum operator $\hat{N} = \hat{N}_a + \hat{N}_c$ and the phase difference operator $\hat{\phi}^- = \hat{\phi}_a - \hat{\phi}_c$. Such state reads [21]

$$|\phi_N^-\rangle_{ac} = \frac{1}{\sqrt{2\pi}}\sum_{j=0}^{N} e^{ij\phi^-}|j\rangle_a|N-j\rangle_c, \tag{15}$$

with ϕ^- is restricted in the windows $\phi_0^- \le \phi^- < \phi_0^- + 2\pi$ with ϕ_0^- is an arbitrary real number. We clearly write the Eq. (14) as

$$|\Phi_b\rangle = \frac{P^{-1/2}}{\sqrt{2\pi}}\sum_{n=0}^{N} c_{n,q}d_{N-n}e^{-in\phi^-}|n+q\rangle_b, \tag{16}$$

with

$$P = \frac{1}{2\pi}\sum_{n=0}^{N}|c_{n,q}|^2|d_{N-n}|^2. \tag{17}$$

After measurement, A sends to B the number of N and ϕ^- by a classical channel. Using these data, B rotates his phase by the unitary operator $\hat{U} = e^{i(\hat{N}_b - q)\phi^-}$ with $\hat{N}_b$ is the photon number operator of mode b. The state of B becomes

$$|\Phi\rangle = \frac{P^{-1/2}}{\sqrt{2\pi}}\sum_{n=0}^{N} c_{n,q}d_{N-n}|n+q\rangle_b. \tag{18}$$

Then, B transforms the photon number $n+q$ to become $N-n$. The teleportation process is completed. The output state is

$$|\Phi\rangle_{out} = \frac{P^{-1/2}e^{-|\alpha|^2/2}}{\sqrt{2\pi}}\sum_{n=0}^{N} c_{n,q}\frac{\alpha^{N-n}}{\sqrt{(N-n)!}}|N-n\rangle, \tag{19}$$

and in case $q=0$ and $\xi = |\xi|$, the average fidelity is

$$F_{av} = \mathcal{N}_0^2(|\xi|)e^{-2|\alpha|^2}\sum_{N=0}^{\infty}\left|\sum_{n=0}^{N}\frac{|\xi|^n|\alpha|^{2N-2n}}{(N-n)!n!}\right|^2. \tag{20}$$

4. Results and discussions

Using the Eqs. (10) and (20), we discus about enhancing the average fidelity of the quantum teleportation process via pair coherent states as the entanglement sources in two difference protocols. Based on Eq. (10), it is easy to see that the average fidelity in the orthogonal quadrature components protocol does not depend on the amplitude of the input state $|\alpha|$, but depends on the amplitude of the pair coherent states $|\xi|$ and q. The figure 1 plots the dependence of the F_{av} as a function of $|\xi|$ and q corresponding to $q = 0, 1, 3$ and 6, respectively. The difference between our protocol and Gabris-Agarwal protocol [11] is that while the receiver in our protocol performs a photon number shifting by using operator in Eq. (7) and the receiver in Gabris-Agarwal protocol is not. That makes the average fidelity in the quantum teleportation process depends on q. Note that the case $q = 0$, the F_{av} in Eq. (10) reduces to the Gabris-Agarwal protocol, in which the highest average fidelity is 0.75884. It shows in figure 1 that the teleportation process is successful, and the average fidelity is more improved by increasing of q. If we add a photon number shifting in the quantum teleportation process, maximum of the average fidelity can approach to the unit by choosing high value of charge q. When a receiver performs a photon number shifting before using a displacement operator in the process to construct the output state, the average fidelity is enhanced by increasing of q, even F_{av} can approach to the unit if q becomes very big.

Based on Eq. (20), we can see that the average fidelity in the measuring of photon number sum and phase difference protocol depends on both the amplitudes of the input state $|\alpha|$ and the pair coherent states $|\xi|$. The figure 2 plots the dependence of the F_{av} as a function of $|\xi|$ and $|\alpha|$ corresponding to $|\alpha| = 0.5$, 1 and 2, respectively. It shows that for giving the values of $|\alpha|$, the quantum teleportation process is also successful, and the average fidelity is more increased by increasing of the $|\xi|$. As previous discussion, the maximum of the average fidelity in the quantum teleportation process in the Gabris-Agarwal protocol [11] is only 0.75884. But in the measuring of photon number sum and phase difference protocol, the average fidelity can approach to the

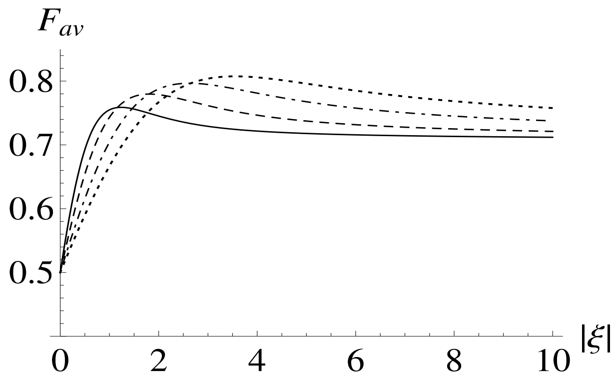

Figure 1. F_{av} as a function of $|\xi|$ and q, from bottom (solid lines) to top (dotted lines) corresponding to $q = 0, 1, 3$ and 6, respectively.

unit by choosing small value of the input state $|\alpha|$ and the big value of the amplitude of the pair coherent states $|\xi|$. For example, if $|\alpha| = 0.5$ and $|\xi| = 15$, the average fidelity of photon number sum and phase difference protocol is 0.99169. That means the quantum teleportation process can be greatly improved and it approaches to maximum value. For given the amplitude of the input state $|\alpha|$, the improvement of F_{av} by increasing of the amplitude of the pair coherent states is achieved, i.e., the average fidelity increases with the increasing of $|\xi|$. With a small value of the input state $|\alpha|$, the average fidelity can approach to the unit in the limit of the big amplitude value of the pair coherent states $|\xi|$.

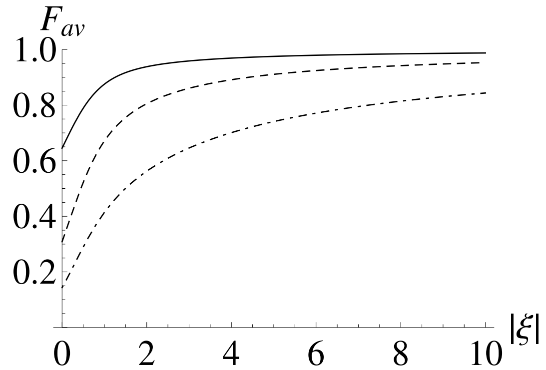

Figure 2. F_{av} as a fuction of $|\xi|$ and $|\alpha|$. From top (solid lines) to bottom (dot-dashed lines) corresponding to $|\alpha| = 0.5$, 1 and 2, respectively.

5. Conclusions

In this paper, we have investigated the quantum teleportation process via the pair coherent states by using two indicator protocols. In the measuring of the orthogonal quadrature components protocol, the average fidelity of the quantum teleportation process does not depend on the amplitude of the input state $|\alpha|$ but depends on the amplitude of the pair coherent states $|\xi|$ and q. In this protocol, F_{av} can approach to the unit if q becomes very big. That result plays an important role in the improvement of quantum teleportation fidelity comparing with the Gabris-Agarwal protocol [11]. In the measuring of the photon number sum and phase difference protocol, the average fidelity of the quantum teleportation process depends on both the amplitudes of the input state $|\alpha|$ and the pair coherent states $|\xi|$. In this protocol, F_{av} can also approach to the unit in the limit of the big amplitude value of the pair coherent states $|\xi|$. The difficulty of this protocol is the failure of the teleportation process when amplitude of teleported state becomes very big. However, those results have indicated that we can base on the relation characteristic between entanglement sources and teleported state to propose conditions for the success of the quantum teleportation process.

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References

[1] Bennett C H, Brassard G, Crepeau C, Jozsa R, Peres A and Wootters W K 1993 *Phys. Rev. Lett.* **70** 1895
[2] Furusawa A *et al* 1998 *Science* **282** 706
[3] Hofmann H F, Ide T and Kobayashi T 2000 *Phys. Rev. A* **62** 062304
[4] Janszky J, Koniorczyk M and Gabris A 2001 *Phys. Rev. A* **64** 034302
[5] Milburn G J and Braunstein S L 1999 *Phys. Rev. A* **60** 937
[6] Cochrane P T, Milburn G J and Munro W J 2000 *Phys. Rev. A* **62** 062307
[7] Karlsson A and Bourennane M 1998 *Phys. Rev. A* **58** 6
[8] An N B 2003 *Phys. Rev. A* **68** 022321
[9] Pirandola S, Eisert J, Weedbrook C, Furusawa A and Braunstein S L 2015 *Nature Photonics* **9**
[10] Opatrny T, Kurizki G and Welsch D G 2000 *Phys. Rev. A* **61** 032302
[11] Gabris A and Agarwal G S 2007 *Int. J. Quantum. Inform.* **5** 17
[12] Hou L L, Jia K H and Chen X F 2016 *Optik* **127**
[13] Li S B, Wu R K, Wang Q M and Xu J B 2004 *Phys. Lett. A* **325**
[14] Hoai N T X and Duc T M 2016 *Int. J. Mod. Phys. B* **30** 1650032
[15] Ulanov A E, Sychev D, Pushkina A A, Fedorov I A and Lvovsky A I 2017 *Phys. Rev. Lett.* **118** 160501
[16] Agarwal G S 1986 *Phys. Rev. Lett.* **57** 827
[17] Dong Y L, Zou X B and Guo G K 2008 *Phys. Lett. A* **372**
[18] Gery C C, Mimih J and Birrittella R 2011 *Phys. Rev. A* **84** 023810
[19] Duc T M, Dinh D H and Dat T Q 2016 *Int. J. Theor. Phys.* **55** 3027
[20] Kurucz Z, Adam P, Kis Z and Janszky J 2005 *Phys. Rev. A* **72** 052315
[21] Luis A and Sanchez-Soto L L 1996 *Phy. Rev. A* **51** 1