Superlattices and Microstructures 112 (2017) 353–361

Contents lists available at ScienceDirect

Superlattices and Microstructures

journal homepage: www.elsevier.com/locate/superlattices

Theoretical investigation of magnetoresistivity oscillations modulated by a terahertz field in quantum wells with parabolic potential

Bui Dinh Hoi

Department of Physics, University of Education, Hue University, 34 - Le Loi St, Hue City, Viet Nam

ARTICLE INFO

Article history:
Received 9 July 2017
Received in revised form 24 September 2017
Accepted 24 September 2017
Available online 26 September 2017

Keywords:
Magnetoresistivity
Quantum well
Electron - phonon interaction
Terahertz field

ABSTRACT

The magnetoresistivity (MR) in a parabolic quantum well (PQW), subjected to a crossed dc electric field and magnetic field, modulated by a terahertz field (TF), is theoretically calculated. The electron - acoustic phonon interaction is taken into account at low temperatures. In the case of absence of the TF, the Shubnikov - de Haas oscillations are observed. The temperature dependence of the relative amplitude of these oscillations is in good agreement with previous theories and experiments in some two-dimensional electron systems. In the presence of the TF, there exist the oscillations in the MR which are similar to those observed experimentally in some two-dimensional electron systems. The amplitude of these oscillations increases with increasing the TF amplitude (intensity).

1. Introduction

The discoveries of the quantum Hall and Shubnikov - de Haas (SdH) effects open up a large number of applications in materials science. SdH oscillations in the magnetoresistance can be used in experiments to extract basic information of materials such as the carrier concentration, the effective mass, the momentum relaxation time, the carrier mobility, and so on [1]. For example, in the works [2,3] the authors investigated experimentally the dependence of the magnetoresistance on the temperature and used the temperature-dependent relative amplitude of SdH oscillations to determine the electron effective mass. Theoretically, the dependence of the relative amplitude of these oscillations on temperature had been studied before in a two-dimensional electron gas (2DEG) [1,2] utilising a connection between the conductivity and the density of states, which is a function of the single-relaxation time. The results showed that the relative amplitude at a fixed magnetic field decreases as the temperature increases.

When 2DEGs are subjected simultaneously to a magnetic field and a microwave, one can observe the so-called microwave-induced magnetoresistance oscillations [4–11]. It has been shown that the occurrence of maximum peaks of the magnetoresistance is governed by the ratio of the cyclotron and the electromagnetic wave (EMW) frequency [4–11]. However, little theoretical discussion has been made thus for. On the other hand, the Boltzmann equation was applied to study theoretically the Hall effect in three-dimensional (3D) materials under the influence of EMWs [12–17]. The odd and even properties of the magnetoresistance were also considered. The problem is that in a strong quantum limit (high magnetic field, low temperature), the Boltzmann equation is no longer valid. Then, a quantum theory is necessary to study these effects at quantum conditions [18]. In this work, based on the Hamiltonian of electrons in a parabolic quantum well (PQW), subjected to a crossed

E-mail address: buidinhhoi@hueuni.edu.vn.

https://doi.org/10.1016/j.spmi.2017.09.044

dc electric field and magnetic field in the presence of a terahertz EMW, we derive a quantum transport equation for electron. From this equation, the magnetoresistivity (MR) is calculated for the electron - acoustic phonon scattering at low temperatures and one-photon absorption/emission limit. The paper is organised as follow. In the next section, we describe our theoretical model and the brief derivation of the quantum transport equation for electrons. The calculation of the MR is presented briefly in Sec. 3. Numerical results and discussion are given in Sec. 4. Finally, remarks and conclusions are given briefly in Sec. 5.

2. Theoretical model and transport equation for electrons

We consider the transport of an electron gas in a quantum well structure, in which a one-dimensional electron gas is confined in a heterostructure by a parabolic potential $U(z)$ along the z-direction, and electron motions are free along two directions (assumed the $(x-y)$ plane). A static magnetic field $\overrightarrow{B}$ is applied in the z-direction and a dc electric field $\overrightarrow{E}_1$ is applied in the x-direction. Then, the one-electron Hamiltonian (h^0), its normalised eigenfunctions ($|\xi\rangle$), and the eigenvalues (ε_ξ) in the Landau gauge for the vector potential $\overrightarrow{A}=(0,Bx,0)$ are, respectively, given by Refs. [19–21]

$$h^0=\frac{\left(\overrightarrow{p}+e\overrightarrow{A}\right)^2}{2m_e}+U(z)+eE_1x, \tag{1}$$

$$|N,n,k_y\rangle=\frac{1}{\sqrt{L_y}}\exp(ik_yy)\phi_N(x-x_0)\psi_n(z), \tag{2}$$

$$\varepsilon_{N,n}\left(\overrightarrow{k}_y\right)=\left(N+\frac{1}{2}\right)\hbar\omega_c+\varepsilon_n-\hbar v_dk_y+\frac{1}{2}m_ev_d^2;\ N=0,1,2,\ldots, \tag{3}$$

where e and m_e are the charge and the effective mass of a conduction electron, respectively, $\overrightarrow{p}$ is its momentum operator, N is the Landau level index and n denotes level quantisation in z-direction, $v_d=E_1/B$ is the drift velocity and $\omega_c=eB/m_e$ is the cyclotron frequency. Also, ϕ_N represents harmonic oscillator wave functions, centered at $x_0=-\ell_B^2(k_y-m_ev_d/\hbar)$ where $\ell_B=\sqrt{\hbar/(m_e\omega_c)}$ is the radius of the Landau orbit in the $(x-y)$ plane. Here, $\overrightarrow{k}_y$ and L_y are the wave vector and normalisation length in the y-direction, respectively. For a parabolic well given by $U(z)=m_e\omega_z^2z^2/2$ with the characteristic frequency ω_z of the confinement potential, the one-electron normalised eigenfunctions and the corresponding eigenvalues in the conduction band are, respectively, given by Ref. [21]

$$\psi_n(z)\equiv|n\rangle=\left(\frac{1}{2^nn!\sqrt{\pi}\ell_z}\right)^{1/2}\exp\left(-\frac{z^2}{2\ell_z^2}\right)H_n\left(\frac{z}{\ell_z}\right), \tag{4}$$

$$\varepsilon_n=\left(n+\frac{1}{2}\right)\hbar\omega_z,\quad n=0,1,2,\ldots, \tag{5}$$

where $H_n(x)$ is the n^{th} Hermite polynomial and ℓ_z is $\sqrt{\hbar/(m\omega_z)}$.

When a terahertz field (TF) with the electric field vector $\overrightarrow{E}=(0,E_0\sin\omega t,0)$ (E_0 and ω are the amplitude and the frequency, respectively) propagates in the structure, the Hamiltonian of electron - phonon system, in the second quantisation representation, can be written similarly to the ones obtained in Refs. [22–24]. Then, one can obtain an equation for the time-dependent electron distribution function as

$$\begin{aligned}\frac{\partial f_{N,n,k_y}}{\partial t}=-\frac{2\pi}{\hbar}\sum_{N',n',\overrightarrow{q}}|D_{N,n,N',n'}(\overrightarrow{q})|^2\sum_{s=-\infty}^{+\infty}J_s^2\left(\frac{\lambda}{\omega}\right)\Big\{\Big[f_{N',n',k_y+q_y}\left(N_{\overrightarrow{q}}+1\right)-f_{N,n,k_y}N_{\overrightarrow{q}}\Big]\delta\Big(\varepsilon_{N',n'}\left(\overrightarrow{k}_y+\overrightarrow{q}_y\right)-\varepsilon_{N,n}\left(\overrightarrow{k}_y\right)\\-\hbar\omega_{\overrightarrow{q}}-s\hbar\omega\Big)+\Big[f_{N',n',k_y-q_y}N_{\overrightarrow{q}}-f_{N,n,k_y}\left(N_{\overrightarrow{q}}+1\right)\Big]\delta\Big(\varepsilon_{N',n'}\left(\overrightarrow{k}_y-\overrightarrow{q}_y\right)-\varepsilon_{N,n}\left(\overrightarrow{k}_y\right)+\hbar\omega_{\overrightarrow{q}}-s\hbar\omega\Big)\Big\},\end{aligned} \tag{6}$$

where $\lambda=eE_0q_y/(m_e\omega)$, $N_{\overrightarrow{q}}$ is the equilibrium distribution function of phonons, $\hbar\omega_{\overrightarrow{q}}$ is the energy of a phonon with the frequency $\omega_{\overrightarrow{q}}$ and the wave vector $\overrightarrow{q}=(q_x,q_y,q_z)$, $J_s(x)$ is the s^{th}-order Bessel function of argument x, and [19,25]

$$|D_{N,n,N',n'}(\overrightarrow{q})|^2 = |C_{\overrightarrow{q}}|^2 |I_{n,n'}(\pm q_z)|^2 |J_{N,N'}(u)|^2, \tag{7}$$

where $C_{\overrightarrow{q}}$ is the electron-phonon interaction constant which depends on the mechanism of electron-phonon interaction; the term $I_{n,n'}(\pm q_z)$ is called the electron form factor and defined by

$$I_{n,n'}(\pm q_z) = \langle n \left| e^{\pm i q_z z} \right| n' \rangle, \tag{8}$$

and,

$$\left| J_{N,N'}(u) \right|^2 = (N_{\min}!/N_{\max}!) e^{-u} u^{N_{\max}-N_{\min}} \left[L_{N_{\min}}^{N_{\max}-N_{\min}}(u) \right]^2 \tag{9}$$

with $N_{\min} = \min\{N, N'\}$, $N_{\max} = \max\{N, N'\}$, $L_M^N(x)$ being the associated Laguerre polynomial, and $u = \ell_B^2 (q_x^2 + q_y^2)/2$.

In the single (constant) scattering time approximation, the equation for the particle number can be also written classically as

$$\frac{\partial f_{N,n,k_y}}{\partial t} - \left(e\overrightarrow{E}_1 + \hbar\omega_c \left[\overrightarrow{k}_y \wedge \overrightarrow{h} \right] \right) \frac{\partial f_{N,n,k_y}}{\hbar \partial \overrightarrow{k}_y} = -\frac{f_{N,n,k_y} - f_0}{\tau}, \tag{10}$$

where $\overrightarrow{h} = \overrightarrow{B}/B$ is the unit vector along the direction of the magnetic field, the notation '$\wedge$' represents the cross product (vector product), f_0 is the equilibrium distribution function of electrons, and τ is the electron momentum relaxation time, which is assumed to be constant in this calculation. Combining Eq. (6) and Eq. (10), we have

$$\begin{aligned} -\left(e\overrightarrow{E}_1 + \hbar\omega_c \left[\overrightarrow{k}_y \wedge \overrightarrow{h} \right] \right) \frac{\partial f_{N,n,k_y}}{\hbar \partial \overrightarrow{k}_y} = & -\frac{f_{N,n,k_y} - f_0}{\tau} + \frac{2\pi}{\hbar} \sum_{N',n',\overrightarrow{q}} \left| D_{N,n,N',n'}(\overrightarrow{q}) \right|^2 \sum_{s=-\infty}^{+\infty} J_s^2\left(\frac{\lambda}{\omega}\right) \left\{ \left[f_{N',n',k_y+q_y} \left(N_{\overrightarrow{q}} + 1 \right) \right. \right. \\ & \left. - f_{N,n,k_y} N_{\overrightarrow{q}} \right] \delta\left(\varepsilon_{N',n'}\left(\overrightarrow{k}_y + \overrightarrow{q}_y \right) - \varepsilon_{N,n}\left(\overrightarrow{k}_y \right) - \hbar\omega_{\overrightarrow{q}} - s\hbar\omega \right) + \left[f_{N',n',k_y-q_y} N_{\overrightarrow{q}} \right. \\ & \left. \left. - f_{N,n} \left(N_{\overrightarrow{q}} + 1 \right) \right] \delta\left(\varepsilon_{N',n'}\left(\overrightarrow{k}_y - \overrightarrow{q}_y \right) - \varepsilon_{N,n}\left(\overrightarrow{k}_y \right) + \hbar\omega_{\overrightarrow{q}} - s\hbar\omega \right) \right\}. \end{aligned} \tag{11}$$

Equation (11) is the quantum kinetic equation for electrons interacting with phonons. It is fairly general and can be applied for any kind of phonon including acoustic phonon, polar optical phonon, non-polar optical phonon, etc. In the following, we will use this equation to derive analytical expression of the MR in the present model.

3. Analytical expression for the magnetoresistivity

When an electron absorbs a photon, it acquires an energy $\hbar\omega$. In the presence of disorder the excited electrons can be scattered by impurities or phonons. It has been shown that at low temperatures, the longitudinal optical phonon scattering becomes negligible and the main source of energy relaxation is acoustic phonon scattering [3]. In this calculation we only consider the electron - acoustic phonon scattering at low temperatures and limit the calculation to the cases of $s = -1, 0, 1$. This means that only one-photon (absorption/emission) processes are taken into account. If the electron - acoustic phonon scattering is elastic, the acoustic phonon energy in Eq. (11) can be neglected [26]. This is reasonable because when the magnetic field and the TF frequency are relatively high, the acoustic phonon energy are much smaller than the cyclotron and photon energy. If we multiply both sides of Eq. (11) by $(e\hbar/m_e)\overrightarrow{k}_y \delta(\varepsilon - \varepsilon_{N,n}(\overrightarrow{k}_y))$ and carry out the summations over N, n, and k_y, we can write out the equation for the partial current density $\overrightarrow{j}(\varepsilon)$ (the current caused by electrons that have energy of ε) as

$$\frac{\overrightarrow{j}(\varepsilon)}{\tau} + \omega_c \left[\overrightarrow{h} \wedge \overrightarrow{j}(\varepsilon) \right] = \overrightarrow{Q}(\varepsilon) + \overrightarrow{S}(\varepsilon), \tag{12}$$

where

$$\overrightarrow{Q}(\varepsilon) = -\frac{e}{m_e} \sum_{N,n,k_y} \overrightarrow{k}_y \left(\overrightarrow{F} \frac{\partial f_{N,n,k_y}}{\partial \overrightarrow{k}_y} \right) \delta\left(\varepsilon - \varepsilon_{N,n}\left(\overrightarrow{k}_y \right) \right), \quad \overrightarrow{F} = e\overrightarrow{E}_1, \tag{13}$$

and

$$\overrightarrow{S}(\varepsilon)=\frac{4\pi e}{m_{\rm e}}\sum_{N,n,N',n'}\sum_{\overrightarrow{q},k_y}\left|D_{N,n,N',n'}(\overrightarrow{q})\right|^2 N_{\overrightarrow{q}}\overrightarrow{k}_y\left\{\left[f_{N',n',k_y+q_y}-f_{N,n,k_y}\right]\times\left[\left(1-\frac{\lambda^2}{2\omega^2}\right)\delta\left(\varepsilon_{N',n'}\left(\overrightarrow{k}_y+\overrightarrow{q}_y\right)-\varepsilon_{N,n}\left(\overrightarrow{k}_y\right)\right)\right.\right.$$
$$\left.\left.+\frac{\lambda^2}{4\omega^2}\delta\left(\varepsilon_{N',n'}\left(\overrightarrow{k}_y+\overrightarrow{q}_y\right)-\varepsilon_{N,n}\left(\overrightarrow{k}_y\right)+\hbar\omega\right)+\frac{\lambda^2}{4\omega^2}\delta\left(\varepsilon_{N',n'}\left(\overrightarrow{k}_y+\overrightarrow{q}_y\right)-\varepsilon_{N,n}\left(\overrightarrow{k}_y\right)-\hbar\omega\right)\right]\right\}\delta\left(\varepsilon-\varepsilon_{N,n}\left(\overrightarrow{k_y}\right)\right). \tag{14}$$

Solving Eq. (12) (see Appendix) we have

$$\overrightarrow{j}(\varepsilon)=\frac{\tau}{1+\omega_{\rm c}^2\tau^2}\left\{\left(\overrightarrow{Q}(\varepsilon)+\overrightarrow{S}(\varepsilon)\right)-\omega_{\rm c}\tau\left(\left[\overrightarrow{h}\wedge\overrightarrow{Q}(\varepsilon)\right]+\left[\overrightarrow{h}\wedge\overrightarrow{S}(\varepsilon)\right]\right)+\omega_{\rm c}^2\tau^2\left(\overrightarrow{Q}(\varepsilon).\overrightarrow{h}+\overrightarrow{S}(\varepsilon).\overrightarrow{h}\right)\overrightarrow{h}\right\}. \tag{15}$$

The total current density is given by

$$\overrightarrow{J}=\int_0^\infty\overrightarrow{j}(\varepsilon)d\varepsilon. \tag{16}$$

If the temperature is low enough, the electrons system is degenerate and the electron distribution function is assumed to be the Heaviside step function. In addition, $N_{\overrightarrow{q}}=k_{\rm B}T/(\hbar\omega_{\overrightarrow{q}})=k_{\rm B}T/(\hbar v_{\rm s}q)=(\beta\hbar v_{\rm s}q)^{-1}$ with $\beta=(k_{\rm B}T)^{-1}$, and

$$|C_{\overrightarrow{q}}|^2=\frac{\hbar E_{\rm d}^2 q}{2\rho v_{\rm s}V_0}, \tag{17}$$

where $k_{\rm B}$, $v_{\rm s}$, $E_{\rm d}$, ρ, and V_0 are the Boltzmann constant, the sound velocity in the material, the acoustic deformation potential, the mass density, and the normalisation volume of specimen, respectively. Inserting Eq. (15) into Eq. (16) and performing some manipulation, we obtain the expression for the conductivity tensor, σ_{im}, as

$$\sigma_{im}=\frac{\tau}{1+\omega_{\rm c}^2\tau^2}\left(\delta_{ij}-\omega_{\rm c}\tau\varepsilon_{ijk}h_k+\omega_{\rm c}^2\tau^2h_ih_j\right)\times\left\{a\delta_{jm}+\frac{be}{m_{\rm e}}\frac{\tau}{1+\omega_{\rm c}^2\tau^2}\delta_{jl}\left(\delta_{lm}-\omega_{\rm c}\tau\varepsilon_{lmp}h_p+\omega_{\rm c}^2\tau^2h_lh_m\right)\right\}, \tag{18}$$

where δ_{ij} is the Kronecker delta, ε_{ijk} being the antisymmetric Levi - Civita tensor, the Latin symbols i,j,k,l,m,p stand for the components x,y,z of the Cartesian coordinates,

$$a=\frac{n_0e^2L_y}{2\pi m_{\rm e}\alpha}\sum_{N,n}(\varepsilon_{N,n}-\varepsilon_{\rm F}), \tag{19}$$

$$b=\frac{4\pi e\hbar}{m_{\rm e}}\{b_1+b_2+b_3+b_4\}, \tag{20}$$

$$b_1=\gamma\left(\frac{eB\overline{\ell}}{\hbar}\right)\left\{1+2\sum_{\eta=1}^{+\infty}(-1)^\eta\exp\left(-\frac{2\pi\eta\Gamma}{\hbar\omega_{\rm c}}\right)\cos(2\pi\eta\overline{n}_1)\right\}, \tag{21}$$

$$b_2=-\frac{\gamma\vartheta}{2}\left(\frac{eB\overline{\ell}}{\hbar}\right)^3\left[1+2\sum_{\eta=1}^{+\infty}(-1)^\eta\exp\left(-\frac{2\pi\eta\Gamma}{\hbar\omega_{\rm c}}\right)\cos(2\pi\eta\overline{n}_1)\right], \tag{22}$$

$$b_3=\frac{\gamma\vartheta}{4}\left(\frac{eB\overline{\ell}}{\hbar}\right)^3\left[1+2\sum_{\eta=1}^{+\infty}(-1)^\eta\exp\left(-\frac{2\pi\eta\Gamma}{\hbar\omega_{\rm c}}\right)\cos(2\pi\eta\overline{n}_2)\right], \tag{23}$$

$$b_4=\frac{\gamma\vartheta}{4}\left(\frac{eB\overline{\ell}}{\hbar}\right)^3\left[1+2\sum_{\eta=1}^{+\infty}(-1)^\eta\exp\left(-\frac{2\pi\eta\Gamma}{\hbar\omega_{\rm c}}\right)\cos(2\pi\eta\overline{n}_3)\right], \tag{24}$$

$$\overline{n}_1 = \frac{(n-n')\hbar\omega_z + eE_1\overline{\ell}}{\hbar\omega_c}, \tag{25}$$

$$\overline{n}_2 = \overline{n}_1 - \frac{\omega}{\omega_c}, \quad \overline{n}_3 = \overline{n}_1 + \frac{\omega}{\omega_c}, \tag{26}$$

ε_F and n_0 are, respectively, the Fermi level and the electron density, $\gamma = n_0 C L_y I(n,n')(\varepsilon_{N,n} - \varepsilon_F)/(8\pi^3\beta v_s \omega_c \hbar^2 \alpha^2 \ell_B^2)$, $C = \hbar E_d^2/(2\rho v_s)$, $\beta = 1/(k_B T)$, $\alpha = \hbar v_d$, $\vartheta = e^2 E_0^2/(m_e^2\omega^4)$, $\overline{\ell} = (\sqrt{N+1/2} + \sqrt{N+1+1/2})\ell_B/2$, $\varepsilon_{N,n} = (N+1/2)\hbar\omega_c + (n+1/2)\hbar\omega_z + m_e v_d^2/2$,

$$I(n,n') = \int_{-\infty}^{+\infty} \left|I_{n,n'}(\pm q_z)\right|^2 dq_z, \tag{27}$$

and Γ is the damping factor associated with the momentum relaxation time, τ, by $\Gamma \approx \hbar/\tau$ [26]. The MR is obtained from the conductivity tensor by the formula

$$\rho_{xx} = \frac{\sigma_{xx}}{\sigma_{xx}^2 + \sigma_{yx}^2}, \tag{28}$$

where σ_{xx} and σ_{yx} are given by Eq. (18).

Equation (28) shows the dependence of the MR on the external fields, including the TF. It is obtained for arbitrary values of the indices N, n, N', n' and looks complicated. The integral (27) was calculated analytically in detail for several transitions between electronic subbands, such as $n = 0, n' = 1$ [33,34]. In this calculation, it will be computationally evaluated. If the well potential is not parabolic (for instance, rectangular or triangle) or has a finite height, the wavefunction and the energy spectrum in the confinement direction no longer take the forms (4) and (5). This leads to the changes of the electron form factor, the transition rate and the magnetoresistivity. In the following, we will give a deeper insight to the above results by carrying out a numerical evaluation and a graphic consideration.

4. Numerical results and discussion

To deduce some physical conclusions of the above results, in this section we choose GaAs/$Al_{0.32}Ga_{0.68}As$ PQW to carry out numerical calculations of the MR. The parameters of the PQW are as follows [27,28]: $\varepsilon_F = 0.115 \times 10^{-18}$ J, $E_d = 13.5$ eV, $\rho = 5320$ kg.m^{-3}, $v_s = 5378$ m.s^{-1}, $m_e = 0.067 \times m_0$ (m_0 is the free electron mass). We also take $\tau = 10^{-12}$ s, $L_x = L_y = 100$ nm and only consider the transitions: $N = 0, N' = 1, n = 0, n' = 1$ (the lowest and the first-excited levels). Two separated cases are considered: absence and presence of the TF.

Fig. 1 shows the dependence of the MR on the magnetic field at different values of the temperature when the TF is absent. We can see the appearance of the typical SdH oscillations whose period is proportional to $1/B$, which is confirmed in Fig. 2. It

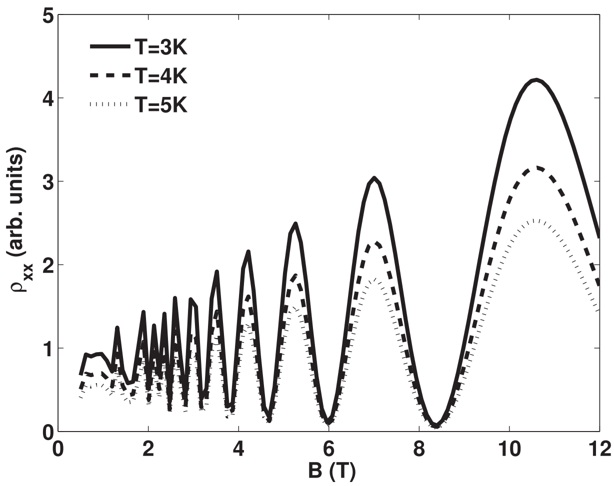

Fig. 1. The MR as functions of the magnetic field at different values of the temperature in the case of absence of the terahertz field ($E_0 = 0$). Here, $E_1 = 5 \times 10^2$ V/m and $\omega_z = 5.5 \times 10^{13}$ s^{-1}.

is seen that the period of the oscillations does not depend on the temperature. Also, the oscillation amplitude at a fixed magnetic field decreases with increasing the temperature. To obtain the law of the temperature dependence of the MR, we consider the relative amplitude of the SdH oscillations. The relative amplitude is defined as the ratio of amplitudes of the oscillation peaks at a fixed magnetic field, B_n, and at temperatures T and T_0, denoted by $A(T,B_n)/A(T_0,B_n)$. In Fig. 3, we show the relative amplitude for $B_n = 2.126$ T and $T_0 = 1.5$ K. The results obtained in other works are also plotted in this figure to make a comparison. It can be seen that the temperature-dependent relative amplitude in this study is in good agreement with those obtained experimentally by N. Balkan and his coworkers in the GaAs/AlGaAs multiple-quantum-wells. The temperature dependence of the relative amplitude was also theoretically studied in the absence of the EMW by Linke [1] and Balkan [2]. From Fig. 3, we can see a good accordance between our result and the formulae in Refs. [1] and [2].

The effect of the material structure on the MR is also investigated in Fig. 4 where we plot the MR at different values of the confinement frequency of the PQW. We can see very clearly that at low confinement frequencies, i.e., weak confinement, the MR oscillation is suppressed, and as the confinement frequency increases the oscillations become more evident. This behaviour is reasonable because the SdH oscillations can be observed only in low-dimensional materials. When the confinement is weak, electrons in the PQW behave as a 3D system, hence, the MR oscillations disappear.

We now consider the effect of the TF on the MR oscillations. In Fig. 5, we plot the MR versus the ratio ω/ω_c at a fixed ω for two cases: absence (the dashed curve) and presence of the TF with $E_0 = 2 \times 10^5$ V/m and $\omega = 7 \times 10^{12}$ Hz (the solid curve). It is seen that the TF makes the SdH oscillation broken down, especially at strong magnetic field, and a beat-like oscillation occurs in this case. We also see that the amplitude of the oscillation remains unchanged at positions where the TF frequency is equal to the cyclotron frequency multiplying by an integer. In contrast, it decreases most at positions where the TF frequency

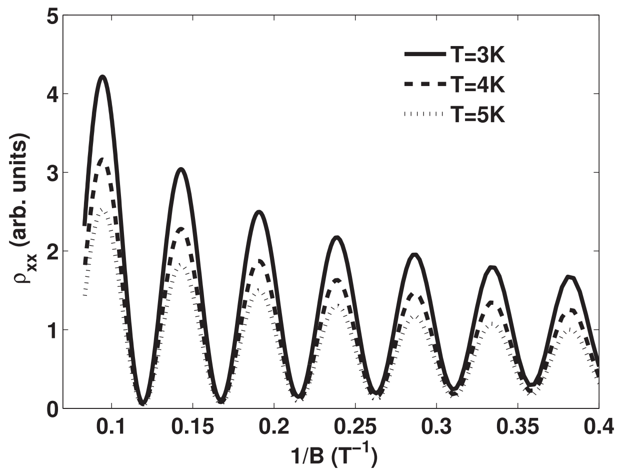

Fig. 2. The MR versus the inversion of magnetic field at different values of the temperature in the case of absence of the terahertz field ($E_0 = 0$). Here, $E_1 = 5 \times 10^2$ V/m and $\omega_z = 5.5 \times 10^{13}$ s^{-1}.

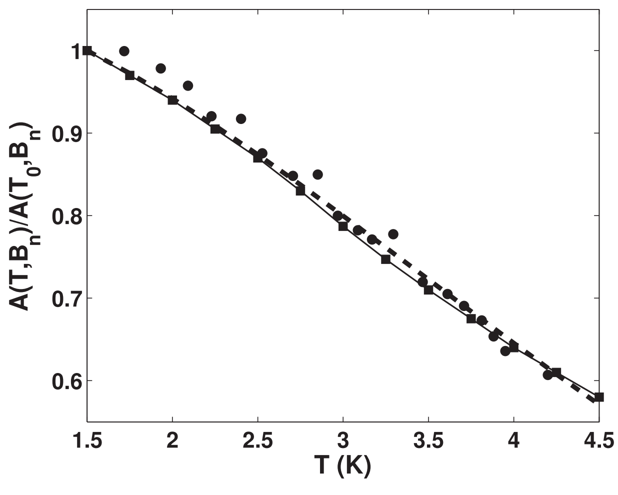

Fig. 3. The relative amplitude $A(T,B_n)/A(T_0,B_n)$ versus temperature. The full squares are our calculation, the full circles are experimental measurements for GaAs/$Al_{0.32}Ga_{0.68}$ As multiple-quantum-wells from Ref. [2], and the dashed curve is the theory in Ref. [1].

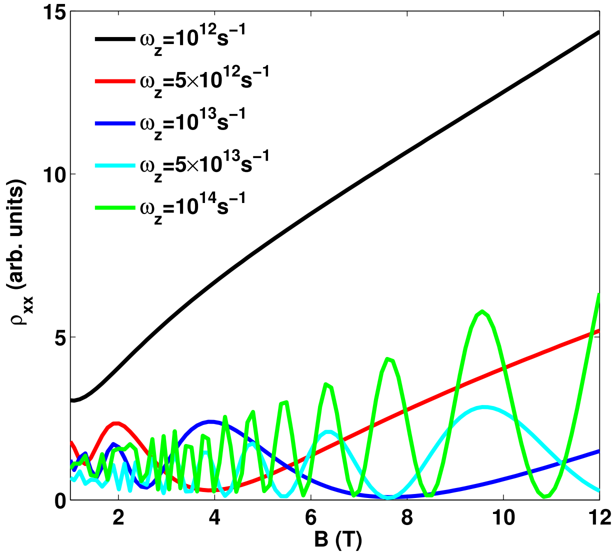

Fig. 4. The MR versus the magnetic field in the absence of the TF at different values of the confinement frequency of the PQW. Here, $T = 4$ K and $E_1 = 5 \times 10^2$ V.m^{-1}.

is equal to the half-integer of the cyclotron frequency. Moreover, at large magnetic field region the oscillations in both cases are apparently the same. This behaviour has been observed in a GaAS-based 2DEG in the presence of a microwave at high frequencies ($\omega/2\pi \sim 280$ GHz) [9].

To show the effect of the TF amplitude, E_0, on the MR, in Fig. 6 we plot the MR on the ratio ω/ω_c for the same system at different values of E_0. Here, ω_c is fixed ($\omega_c \approx 7.8835 \times 10^{12}$ s^{-1} for $B = 3$ T). It is seen that the amplitude of MR oscillations depends strongly on the TF amplitude. As the TF amplitude increases, the oscillation amplitude increases. This means that the larger the intensity of the TF, the greater its influence on the MR. Similarly to Fig. 5, we can see in Fig. 6 the MR maxima at $\omega/\omega_c = 1, 2, 3, \ldots$ and the minima at $\omega/\omega_c = 3/2, 5/2, 7/2, \ldots$. Also, the positions of these maxima and minima do not depend on the TF amplitude. This property is similar to those obtained experimentally in AlGaAs/GaAs quantum wells [5,6,8] where the MR was modulated by a microwave radiation with frequency $\omega/2\pi$ from 27 GHz to 150 GHz. Theoretically, it was also explained by Ryzhii and Vyurkov [29] considering the scattering of electrons by acoustic piezoelectric phonons accompanied by the absorption of microwave photons, and by Raichev [30] using the Kubo formula. These MR oscillations induced by an ac field have been studied in details both theoretically [10,11] and experimentally [4–8]. Moreover, the maxima of the MR show the cyclotron resonant behaviour of electron when the photon energy equals multiple-integer of the cyclotron energy. A

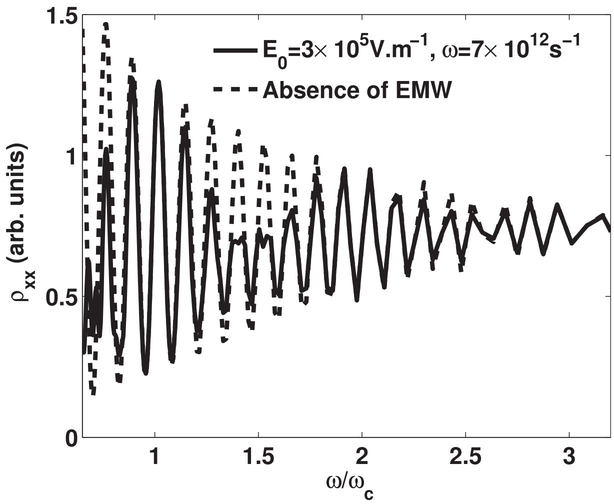

Fig. 5. Dependencies of the MR on the ratio ω/ω_c with fixed ω for two cases: presence (solid curve) and absence (dashed curve) of the TF. Here, $T = 4$ K, $E_1 = 5 \times 10^2$ V/m, and $\omega_z = 5.5 \times 10^{13}$ s^{-1}.

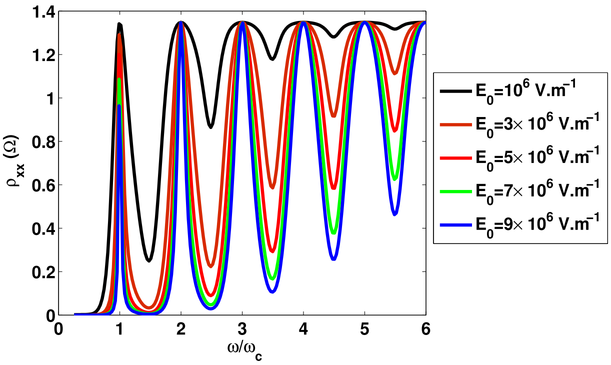

Fig. 6. Dependencies of the MR on the ratio of the TF frequency and cyclotron frequency at different values of the TF amplitude. Here, $B = 3$ T ($\omega_c = 7.8835 \times 10^{12}$ s^{-1}), $T = 4$ K, $E_1 = 5 \times 10^2$ V.m^{-1}, and $\omega_z = 5.5 \times 10^{13}$ s^{-1}.

review of the very first observations of the cyclotron resonance in a n-channel inversion layer on the Si(100) can be found in the work by Ando, Fowler, and Stern [31]. The cyclotron resonance has been studied widely in various materials including parabolic quantum wells [32–34].

5. Conclusions

So far, we have investigated the MR in a PQW, subjected to a crossed dc electric field and magnetic field under the influence of a TF. The electron - acoustic phonon interaction mechanism is adopted to calculate the conductivity tensor as well as the MR at low temperature. Electron gas is assumed to be degenerate. The analytical results are applied for GaAs/$Al_{0.32}Ga_{0.68}As$ PQW to clarify physical meanings by carrying out numerical calculations. When the TF is absent we can see the SdH oscillations in the MR. The period of these oscillations does not depend on the temperature whereas the amplitude decreases with increasing temperature. The dependence of the relative amplitude of these oscillations is in good agreement with the results obtained previously by other methods in similar structures. As the TF is switched on, there occurs the magnetoresistance oscillations induced by an ac field (TF) which show the beat phenomenon, especially at large magnetic field region. The amplitude of MR oscillations then has maximum values at $\omega/\omega_c = 1, 2, 3, \ldots$ and minimum values at $\omega/\omega_c = 3/2, 5/2, 7/2, \ldots$ In addition, the amplitude of MR oscillations increases as the TF amplitude (intensity) increases.

Appendix

In this appendix, we present some steps to obtain Eq. (15) from Eq. (12) as follows.
Multiplying both sides of Eq. (12) by $\omega_c\tau^2$, we have

$$\omega_c\tau\vec{j} + \omega_c^2\tau^2\left[\vec{h}\wedge\vec{j}\right] = \omega_c\tau^2\vec{Q} + \omega_c\tau^2\vec{S}. \tag{29}$$

Taking the cross product between $\vec{h}$ and both sides of Eq. (29) with the notice that $\vec{h}.\vec{h} = 1$, we have

$$\begin{aligned} &\omega_c\tau\left[\vec{h}\wedge\vec{j}\right] + \omega_c^2\tau^2\left[\vec{h}\left(\vec{h}\cdot\vec{j}\right) - \vec{j}\left(\vec{h}\cdot\vec{h}\right)\right] = \omega_c\tau^2\left[\vec{h}\wedge\vec{Q}\right] + \omega_c\tau^2\left[\vec{h}\wedge\vec{S}\right] \\ \Leftrightarrow &\omega_c\tau\left[\vec{h}\wedge\vec{j}\right] + \omega_c^2\tau^2\left[\left(\vec{h}\cdot\vec{j}\right)\vec{h} - \vec{j}\right] = \omega_c\tau^2\left[\vec{h}\wedge\vec{Q}\right] + \omega_c\tau^2\left[\vec{h}\wedge\vec{S}\right]. \end{aligned} \tag{30}$$

From Eq. (12) we also have

$$\left[\vec{h}\wedge\vec{j}\right] = \frac{1}{\omega_c\tau}\left[\tau\left(\vec{Q} + \vec{S}\right) - \vec{j}\right]. \tag{31}$$

Inserting $[\vec{h}\wedge\vec{j}]$ from Eq. (31) into Eq. (30) we have

$$\tau\left(\vec{Q} + \vec{S}\right) - \left(1 + \omega_c^2\tau^2\right)\vec{j} + \omega_c^2\tau^2\left(\vec{h}\cdot\vec{j}\right)\vec{h} = \omega_c\tau^2\left[\vec{h}\wedge\vec{Q}\right] + \omega_c\tau^2\left[\vec{h}\wedge\vec{S}\right]. \tag{32}$$

Taking the dot product of $\vec{h}$ and both sides of Eq. (12), we have

$$\frac{\vec{h}\cdot\vec{j}}{\tau}+\omega_c\left[\vec{h}\wedge\vec{j}\right]\cdot\vec{h}=\left(\vec{Q}+\vec{S}\right)\cdot\vec{h}. \tag{33}$$

The second term on the left side of Eq. (33) is equal to zero, so

$$\vec{h}\cdot\vec{j}=\tau\left(\vec{Q}+\vec{S}\right)\cdot\vec{h}. \tag{34}$$

Inserting Eq. (34) into Eq. (32) we obtain

$$\tau\left(\vec{Q}+\vec{S}\right)+\omega_c^2\tau^3\left(\left(\vec{Q}+\vec{S}\right)\cdot\vec{h}\right)\vec{h}-\omega_c\tau^2\left[\vec{h}\wedge\vec{Q}\right]-\omega_c\tau^2\left[\vec{h}\wedge\vec{S}\right]=\left(1+\omega_c^2\tau^2\right)\vec{j} \tag{35}$$

or

$$\vec{j}=\frac{\tau}{1+\omega_c^2\tau^2}\left\{\left(\vec{Q}+\vec{S}\right)-\omega_c\tau\left(\left[\vec{h}\wedge\vec{Q}\right]+\left[\vec{h}\wedge\vec{S}\right]\right)+\omega_c^2\tau^2\left(\vec{Q}\cdot\vec{h}+\vec{S}\cdot\vec{h}\right)\vec{h}\right\}. \tag{36}$$

References

[1] H. Linke, P. Omling, P. Ramvall, J. Appl. Phys. 73 (1993) 7533.
[2] N. Balkan, H. Celik, A.J. Vickers, M. Cankurtaran, Phys. Rev. B52 (1995) 17210.
[3] E. Tiras, et al., Superlattices Microstruct. 51 (2012) 733.
[4] M.A. Zudov, R.R. Du, J.A. Simmons, J.L. Reno, Phys. Rev. B64 (2001) 201311 (R).
[5] M.A. Zudov, R.R. Du, L.N. Pfeiffer, K.W. West, Phys. Rev. Lett. 90 (2003) 046807.
[6] C. L. Yang, M. A. Zudov, T. A. Knutilla, R. R. Du, L. N. Pfeiffer, and K. W. West, arXiv: cond-mat/0303472.
[7] A.T. Hatke, M.A. Zudov, L.N. Pfeiffer, K.W. West, Phys. E 42 (2010) 1081.
[8] I.I. Lyapilin, A.E. Patrakov, Low. Temp. Phys. 30 (2004) 834.
[9] X. Lei, S.Y. Lin, Appl. Phys. Lett. 86 (2005) 262101.
[10] J. Dietel, L.I. Glazman, F.W.J. Hekking, F. von Oppen, Phys. Rev. B 71 (2005) 045329.
[11] M. Torres, A. Kunold, J. Phys. Condens. Matter 18 (2006) 4029.
[12] E.M. Epshtein, Fiz. Tekh. Poluprovodn. 10 (1976) 1414 ([Russian]).
[13] E.M. Epshtein, Sov. J. Theor. Phys. Lett. 2 (1976) 234 ([Russian]).
[14] V.L. Malevich, E.M. Epshtein, Fiz. Tverd. Tela 18 (1976) 1286 ([Russian]).
[15] V.L. Malevich, E.M. Epshtein, Izvestiya Vysshikh Uchebnykh Zavedenij, Fizika 2 (1976) 121 ([Russian]).
[16] G.M. Shmelev, G.I. Tsurkan, Nguyen Hong Shon, Fiz. Tekh. Poluprovodn. 15 (1981) 156 ([Russian]).
[17] V.V. Pavlovich, E.M. Epshtein, Fiz. Tekh. Poluprovodn. 11 (1977) 809 ([Russian]).
[18] N.Q. Bau, N.V. Hieu, N.V. Nhan, Superlattices Microstruct. 52 (2012) 921.
[19] P. Vasilopoulos, M. Charbonneau, C.M. Van Vliet, Phys. Rev. B 35 (1987) 1334.
[20] B. Mitra, K.P. Ghatak, Phys. Stat. Sol. (b) 164 (1991) K13.
[21] S.C. Lee, J. Korean Phys. Soc. 51 (2007) 1979.
[22] N.Q. Bau, B.D. Hoi, J. Korean Phys. Soc. 60 (2012) 59.
[23] N.Q. Bau, N.V. Nghia, N.V. Hieu, B.D. Hoi, in: PIERS Proceedings, March 25-28, Taipei, 2013, p. 416.
[24] N.Q. Bau, B.D. Hoi, Int. J. Mod. Phys. B 28 (2014) 1450001.
[25] M.P. Chaubey, C.M.V. Vliet, Phys. Rev. B 33 (1986) 5617.
[26] P. Vasilopoulos, Phys. Rev. B 33 (1986) 8587.
[27] B.K. Ridley, Quantum Processes in Semiconductors, Clarendon Press, Oxford, 1993.
[28] J. Singh, Physics of Semiconductors and Their Heterostructures, McGraw-Hill, Singapore, 1993.
[29] V. Ryzhii, V. Vyurkov, Phys. Rev. B 68 (2003) 165406.
[30] O.E. Raichev, Phys. Rev. B 78 (2008) 125304.
[31] T. Ando, A.B. Fowler, F. Stern, Rev. Mod. Phys. 54 (1982) 437.
[32] Huynh Vinh Phuc, Nguyen Ngoc Hieu, Le Dinh, Tran Cong Phong, Opt. Commun. 335 (2015) 37.
[33] Huynh Vinh Phuc, Luong Van Tung, Superlattices Microstruct. 71 (2014) 124.
[34] Tran Cong Phong, Huynh Vinh Phuc, Superlattices Microstruct. 83 (2015) 755.