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Joint power allocation and power splitting for MISO SWIPT RSMA systems with energy-constrained users

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Abstract

Simultaneous wireless information and power transfer (SWIPT) has been widely used in multi-input single-output (MISO) systems to transmit information and energy simultaneously from the base station (BS) towards the users. The traditional approach used in the literature is based on space-division multiple access (SDMA) method by using the multi-user linear precoding technique, where the information decoding is done by considering the multi-user interference as noise. Recently, the novel rate-splitting multiple access (RSMA) method, based on partially decode the multi-user interference and partially treat that interference as noise, has been shown to outperform the SDMA method in multi-user MISO systems. Motivated by the superior performance of RSMA, we consider a multi-user MISO SWIPT system applying the RSMA method, where multiple energy-constrained users are equipped with a power-splitting structure to harvest energy and decode information simultaneously. In particular, we investigate the optimal precoders and power-splitting ratios design to minimize the total transmit power at the BS, subject to constraints of the minimum data rate for users, minimum energy harvesting by users, and maximum power at the BS. The proposed solution for the formulated non-convex problem is based on two phases. First, we convert the non-convex problem into bilevel programming where the upper optimization problem is solved by using particle swarm optimization. Second, we propose two algorithms to solve the inner optimization problem based on a semidefinite relaxation method or a successive interference cancellation method. Numerical results show that RSMA achieves significant improvement over SDMA in reducing the total transmit power.

Keywords SWIPT · MISO · Rate-splitting · Energy harvesting · Particle swarm optimization

1 Introduction

Recently, the heterogeneity of fifth-generation (5G) devices, deployments, and promising approaches has encouraged the development of new multiple access techniques for downlink communications systems. The rate-splitting (RS) transmission strategy [1, 2] promises to overcome the aforementioned massive connectivity problem with superior performance, in comparison with state-of-the-art multiple access.

The fundamental ideas behind RS are the transmission of common and private messages and decoding part of the interference at the receiver. In [1], the authors showed that RS achieves a significant improvement in spectral and energy efficiency over conventional approaches based only on private messages. In addition, RS was shown to provide greater performance in scenarios with imperfect channel state information at the transmitter (CSIT).

In general, RS divides each message into a common part and a private part at the transmitter, where the ratios used to divide the messages are parameters that depend on the setup [1]. All the common parts are grouped to create the common message, which is encoded by using a codebook shared by all the users, requiring it to be decoded by all of them. On the other hand, the private parts are encoded using private codebook, which is known by their respective user. Then, linear precoding is used to simultaneously transmit the common message and private messages. At the

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user side, the first step is to decode the common message by treating private messages as noise. Next, the user applies successive interference cancellation (SIC) to remove the common part from the received signal. Subsequently, the user decodes the private message by treating the interference from other private messages as noise. Therefore, by adjusting the message split and the allocated power of the common and private messages, the transmitter has the facility to tradeoff between two extreme cases, i.e. totally consider the interference as noise and fully decode the interference by using SIC.

RS was successfully applied in the literature to develop a novel multiple access framework for downlink communication systems, called rate-splitting multiple access (RSMA) [3]. RSMA is based on linearly precoded RS and uses SIC to decode and remove part of the interference, considering the remaining part of the interference as noise. In the literature, RSMA was shown to outperform traditional multiple access methods like space-division multiple access (SDMA). In SDMA, the common technique used is multi-user linear precoding (MU-LP), where linear precoders generate several beams, each of them allocated a portion of the total transmit power. SDMA face limitations when the channel strengths are too different between the users and when the number of users increases, since it needs more transmit antennas than users to successfully deal with the multi-user interference, where the superiority of RSMA is because its ability to manage the multi-user interference in an efficient manner by a tradeoff between fully decoding the interference and fully treating it as noise [3]. In particular, we refer to the results in the Subsection V.C of [3] considering a multi-user MISO system, where RSMA outperforms SDMA in terms of weighted sum rate when the authors considered a base station (BS) equipped with four antennas, three single antenna users and 5dB channel gain difference between the three users. In addition, the Subsection V.F of [3] presents a scenario with two transmit antennas at the BS and ten single antenna users, where RS exploits the maximum degrees of freedom (limited by the 2 transmit antennas) and shows a significant improvement over SDMA in terms of weighted sum rate considering the same channel model with i.i.d complex Gaussian entries with zero mean and unit variance for all the users.

In more details, let us look at the related works that took the RS approach without energy-constrained users. Joudeh and Clerckx [2] considered multiuser multi-input single-output (MISO) systems that consist of one multi-antenna base station (BS) serving a set of single-antenna users using the RS strategy. They considered the minimum rate maximization problem under a transmit power constraint, and the power minimization problem under a rate constraint. These problems were solved by using the weighted minimum mean squared error (WMMSE) algorithm. In [4], the authors proposed the hierarchical rate splitting (HRS) in a massive multi-input multi-output (MIMO) system with imperfect CSIT, where the precoders are designed to maximize the minimum achievable rate of the common message to the users. In [5], the authors considered the RS strategy in a downlink multiuser MISO system with imperfect CSIT, and investigated the max-min fairness optimization problem under the constraints of transmit power at the BS and the kth user's worst-case achievable rates, where the cutting-set method combined with the WMMSE algorithm were used to solve the optimization problem. In [6], the authors proposed a linearly precoded RS approach to exploit the SIC receiver architecture in a superposed unicast and multicast transmission system. They considered the weighted sum rate (WSR) maximization problem of unicast messages under the constraints of transmit power at the BS and a minimum rate for multicast messages. That problem was solved by using the WMMSE algorithm. In [3], the authors developed the RSMA framework for downlink multi-antenna systems, where they considered a system model composed of a multi-antenna BS serving single-antenna users. The authors investigated the WSR maximization problem under the constraints of transmit power at the BS and QoS for each user. That problem was solved using the WMMSE algorithm. This paper compared the RSMA framework with SDMA and non-orthogonal multiple access (NOMA) schemes. Numerical results showed that RSMA outperforms those state-of-art schemes for both under-loaded and overloaded network regimes and under different user deployments, such as a variety of channel directions and channel strengths. Furthermore, RSMA was claimed as a general framework, with SDMA and NOMA as special cases. It is worth noting that above related works do not consider users with the ability to harvest radio frequency energy.

Simultaneous wireless information and power transfer (SWIPT), which simultaneously provides both data and energy to wireless users, has recently received a lot of attention as an interesting research area [7–9]. The main practical receiver designs in SWIPT are time switching (TS) and power splitting (PS), and we focus on the PS scheme because it was proved to have a larger rate-energy region and better rate-energy tradeoffs, in comparison with the TS scheme [7, 8]. In the PS structure, the receiver splits the incoming signal into two streams, one for information decoding (ID) and the other for energy harvesting (EH), by using a variable power splitting ratio. Furthermore, SWIPT provides an improvement in the spectral efficiency and offers a solution to limited battery life, as well as solving the troubles involving battery replacement [7, 9].

Next, let us look at the related works in conventional SWIPT systems. Shi et al. [10] considered a multiuser MISO downlink system with SWIPT, where a multi-antenna BS sends information and energy simultaneously to several single-antenna users. The authors investigated the total transmit power minimization problem under the constraints of minimum signal-to-interference-plus-noise ratio (SINR) and minimum energy harvesting, and solved this non-convex problem by using the semidefinite relaxation (SDR) technique. An extension to MIMO system consisting of one multi-antenna transmitter, one multi-antenna EH receiver and one multi-antenna ID receiver was considered in [11], where the authors proposed a transmission strategy to achieve several tradeoffs for data rate versus energy transfer. Dong et al. [12] considered an extension to the multiuser MIMO SWIPT system, where they investigated the transmit power minimization problem subject to the constraints of EH and maximum tolerable mean squared error (MSE). The problem was solved by using an iterative algorithm based on semidefinite programming (SDP). SWIPT in MISO multicasting system was investigated in [13] under the scenarios of perfect and imperfect CSIT, where the authors considered the SDR technique to solve the transmit power minimization problem under the constraints of SNR and EH at the receivers. Smart antenna technologies for SWIPT considering single and multiple users were summarized in [14]. The authors investigated MIMO and relaying techniques applied in SWIPT to improve the energy and the spectral efficiency in wireless systems. Xu et al. [15] considered a downlink MISO system with a cooperative SWIPT NOMA protocol, where the first stage is used to receive the message at the cell-edge user and perform SWIPT at the cell-center user, and the second stage is to forward the message to the cell-edge user using the harvested energy in the cell-center user. The authors aim to maximize the data rate at the cell-center user under the constraints of the QoS requirement of the cell-edge user. The solution of the problem is based on the SDR technique and the successive convex approximation (SCA) algorithm. A power beacon assisted SISO SWIPT system was considered in our own previous work [16], where a power beacon co-exists with the BS in the cell to send energy and information to a user. The user implements a PS structure, and three transmit power minimization problems were optimally solved by the Lagrange method and Karush–Kuhn–Tucker optimality conditions. Tuan and Koo [17] considered a multiuser MISO SWIPT cognitive radio system and investigated a multi-objective problem composed from the max-min of the secondary users' EH and the min–max of the primary users' interference power. Their problem was solved by using the SDR technique combined with particle swarm optimization (PSO).

Zhang et al. [18] studied a millimeter wave based ultra dense network composed of ultra dense small cells overlaid on one macrocell, where the base stations have energy harvesting capabilities. The authors focus on user association and transmit power allocation in the downlink, where the objective is to solve the load-aware energy-efficient user association and power optimization problem with the constraints of user scheduling, total power, cross-tier interference, and the number of associated user. The problem is first transformed into a convex with the Lagrangian dual decomposition method over a relaxed version of the original problem, where the solution is based on the Newton–Raphson method and an iterative gradient algorithm. Zhang et al. [19] considered a heterogeneous small cell network, composed of macrocells and small cells, where the single-antenna small BSs are equipped with EH hardware and SWIPT capabilities. With the objective to maximize the total capacity of the small cell with the constraints of maximum transmit power, and cross-tier/co-tier interference, the authors investigated the optimization problems of power allocation and subchannel assignment under incomplete channel state information. The problems are modeled as a non-cooperative game, where Nash equilibrium solutions are obtained based on the Lagrangian dual function and the subgradient method. However, the authors did not consider multiple antenna BSs, and new access methods as RS.

Although SWIPT in a MISO system has been studied in the literature by using the MU-LP technique, as mentioned above, none of the research investigated the performance of the RS strategy in a MISO SWIPT system. Motivated by the superior performance of the RS strategy to improve the achievable rate compared with state-of-art multiple access methods like SDMA, in this paper, we investigate the RS strategy to reduce the required transmit power at the BS subject to energy-constrained users and minimum data rate requirements. In particular, we consider a multi-user MISO SWIPT RSMA system in which one multi-antennaequipped BS sends data to multiple single-antenna users receiving both information and energy simultaneously under a PS structure. We jointly design the precoders at the BS and the PS ratios to minimize the total transmission power under the constraints of a minimum rate for users, minimum EH by users, and maximum power available at the BS. To the best of our knowledge, this problem and the corresponding solution have not been addressed in the literature. Thus, the main contributions of this paper are summarized as follows.

- The problem of minimum total transmit power at the BS is studied subject to the constraints of a minimum rate for users, minimum EH by users, and maximum power available at the BS. To address this, we convert the non-convex problem into bilevel programming with common rates as upper-level variables, and PS ratios and precoders as lower-level variables. The outer optimization problem is addressed with a PSO algorithm, where we provide mathematical limits on the upper-level variables in order to achieve a rapid convergence. The inner optimization problem is reformulated to an SDP problem via the SDR technique, and the Gaussian randomization technique is applied to find approximate rank-1 solutions.

- In order to substantiate the results provided by the SDR approach, we propose an alternative solution to the inner optimization problem based on the SCA algorithm, where the drawback of the initial feasible points is solved by adding slack variables to relax the original constraints, ensuring feasibility, and a penalty to force the slack variables toward zero.
- The performance of the proposed approach is compared with the SDMA and equal PS schemes. The numerical results show that the proposed algorithms based on the RSMA method achieve a considerably lower transmit power at the BS compared with the the SDMA and EPS schemes, where the SDR- and SCA-based approaches provide the same results, with SDR having the lowest complexity of both algorithms.

The rest of this paper is organized as follows. The system model is described in Sect. 2. Section 3 proposes the minimum transmit power problem formulation and provides optimal solutions by using a PSO-based algorithm with the SDR or SCA based schemes. The simulation results are provided in Sect. 4 for performance analysis. Finally, conclusions are made in Sect. 5.

2 System model

We consider a multi-user MISO SWIPT RSMA system consisting of one BS and K energy-constrained users over a given frequency band, as shown in Fig. 1. The BS, equipped with $N > 1$ antennas, sends messages to K single-antenna receivers. On the user side, we apply the power splitting structure to simultaneously harvest the energy and decode the information from the received signal, as shown in Fig. 2. In particular, the received signal is split into two streams, where one stream with PS ratio $\theta_k \in (0, 1)$ is used for ID, and the other with PS ratio $(1 - \theta_k)$ is used for EH. The baseband equivalent channels from the BS to users are denoted as $\mathbf{h}_k \in \mathbb{C}^{Nx1}$. In this paper, we consider that the BS perfectly knows the instantaneous values for channels $\mathbf{h}_k,\ \forall k \in \{1, \ldots, K\}$, where we assume a quasi-static frequency-flat fading environment.

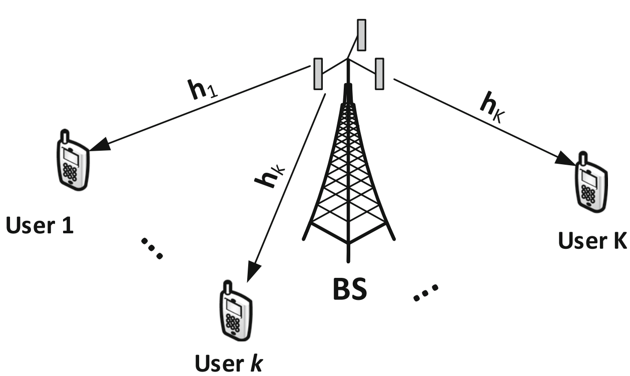

Fig. 1 A multi-user MISO SWIPT RSMA system model

We apply the 1-layer RS strategy [3], where only one SIC is required at the receivers. We denote as m_k the message intended for user k, and we split that message into two parts, $\{m_{k,0}, m_{k,k}\}$, $\forall k$. The messages $m_{1,0}, \ldots, m_{K,0}$ are encoded together into a common stream, s_0, by using a shared codebook; $m_{k,k}$ is the private part of the user k message, and it is encoded into private stream s_k to be decoded by user k only. Note that the common stream, s_0, must be decoded by all users to recover their own messages. We define $\mathbf{p}_0$ and $\mathbf{p}_k \in \mathbb{C}^{Nx1}$ as the precoders for data streams s_0 and s_k, respectively. The complex baseband transmitted signal at the BS can be expressed as $\mathbf{x} = \mathbf{p}_0 s_0 + \sum_{k=1}^{K} \mathbf{p}_k s_k$, where $E\left\{|s_k|^2\right\} = 1,\ \forall k$. We express the average transmit power as $P = \|\mathbf{p}_0\|^2 + \sum_{k=1}^{K} \|\mathbf{p}_k\|^2$. The received baseband signal for the ID module of user k is expressed as

$$y_k^{ID} = \sqrt{\theta_k}\mathbf{h}_k^H\left(\mathbf{p}_0 s_0 + \mathbf{p}_k s_k + \sum_{i=1, i\neq k}^{K} \mathbf{p}_i s_i\right) + v_k, \quad \forall k, \tag{1}$$

where $v_k \sim \mathcal{CN}(0, \delta_k^2)$ is the additive white Gaussian noise (AWGN) at user k, with zero mean and variance δ_k^2, and θ_k is the power splitting ratio.

Figure 2 presents the transmission model using the proposed SWIPT RSMA system. In the ID module, user k first decodes common stream s_0 by treating the interference from all s_k as noise. Then, each user is able to decode part of the interference, since s_0 carries part of the message for

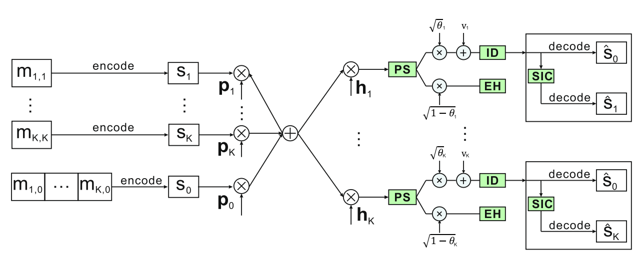

Fig. 2 k-user RS assisted MISO SWIPT transmission model and power splitting structure at the receivers the other users. The SINR of the common stream s_0 at user k is given by

$$\mathrm{SINR}_{0,k} = \frac{\theta_k \left|\mathbf{h}_k^H \mathbf{p}_0\right|^2}{\theta_k \left(\sum_{i=1}^{K} \left|\mathbf{h}_k^H \mathbf{p}_i\right|^2\right) + \delta_k^2}, \quad \forall k. \tag{2}$$

Once s_0 is successful decoded, SIC is applied in the ID module of user k. In particular, the contribution of s_0 is subtracted from y_k^{ID} to decode the user's private stream, s_k. Subsequently, user k decodes private stream s_k by treating the other private streams as noise. Therefore, its SINR reads

$$\mathrm{SINR}_{k} = \frac{\theta_k \left|\mathbf{h}_k^H \mathbf{p}_k\right|^2}{\theta_k \left(\sum_{i=1, i\neq k}^{K} \left|\mathbf{h}_k^H \mathbf{p}_i\right|^2\right) + \delta_k^2}, \quad \forall k. \tag{3}$$

We denote the instantaneous achievable information rates for common stream s_0 and private stream s_k at user k as $R_{0,k} = \log_2\left(1 + \mathrm{SINR}_{0,k}\right)$ and $R_k = \log_2(1 + \mathrm{SINR}_k)$, respectively. To guarantee SIC at the users, we need to successfully decode s_0; hence, the achievable common rate is limited by all $R_{0,k}$, i.e. $R_0 = \min\left\{R_{0,1}, \ldots, R_{0,K}\right\}$. Note that if we use a rate greater than R_0, at least one of the users will not be able to decode common stream s_0. Inspired by the system proposed in [3], we assume that R_0 is shared between the users in such a way that $C_{0,k}$ is the k-th user's portion of the common rate, satisfying $R_0 = \sum_{k=1}^{K} C_{0,k}$. Consequently, the total achievable rate at user k can be defined as $R_{k,tot} = R_k + C_{0,k}$.

Furthermore, the power harvested by the EH module of user k is given by

$$E_k = \eta_k(1 - \theta_k)\left(\left|\mathbf{h}_k^H \mathbf{p}_0\right|^2 + \sum_{i=1}^{K} \left|\mathbf{h}_k^H \mathbf{p}_i\right|^2\right), \quad \forall k, \tag{4}$$

where $\eta_k \in (0, 1]$ is the energy-harvesting efficiency at the EH module of user k.

3 Problem formulation and solutions

3.1 Minimum transmit power problem

Our aim is to minimize the transmit power at the BS subject to the constraints of a minimum rate for users, minimum EH by users, and maximum power available at the BS. The transmit precoding vectors, $\mathbf{p}_0, \{\mathbf{p}_k\}$, the k-th user's portion of the common rates $\left\{C_{0,k}\right\}$, and the receive powersplitting ratios $\{\theta_k\}$ design is accordingly formulated as an optimization problem as follows:

$$\min_{\mathbf{p}_0, \{\mathbf{p}_k, C_{0,k}, \theta_k\}} \|\mathbf{p}_0\|^2 + \sum_{k=1}^{K} \|\mathbf{p}_k\|^2 \quad \text{subject to} \tag{5a}$$

$$C_{0,k} + R_k \geq \gamma_k, \quad \forall k \tag{5b}$$

$$\sum_{i=1}^{K} C_{0,i} \leq R_{0,k}, \quad \forall k \tag{5c}$$

$$C_{0,k} \geq 0, \quad \forall k \tag{5d}$$

$$\mathrm{E_k} \geq \alpha_\mathrm{k}, \quad \forall k \tag{5e}$$

$$\|\mathbf{p}_0\|^2 + \sum_{k=1}^{K} \|\mathbf{p}_k\|^2 \leq P_{\max} \tag{5f}$$

$$0 < \theta_\mathrm{k} < 1, \quad \forall k, \tag{5g}$$

where $P_{\max}$ is the maximum available power at the BS, γ_k is the minimum rate, and α_k is the minimum harvested power required by user k.

Constraint (5c) is to guarantee that s_0 is successfully decoded by the k-th user, constraint (5d) is to ensure real positive values for the k-th user's portion of the common rates, and constraint (5e) represent the minimum EH required by the user. It is worth noting that the PS ratio variables balance the ID and EH performance through the constraints (5b), (5c) and (5e). Problem (5) is challenging to solve, mainly due to the coupling between the power splitting ratios, θ_k, and precoding vectors $\mathbf{p}_0, \{\mathbf{p}_k\}$ in constraints (5b), (5c), and (5e); thus, it cannot be solved directly. In the proposed solution we apply bilevel optimization to solve the Problem (5). In bilevel optimization, one problem is embedded within another, defined as inner optimization problem and outer optimization problem, respectively. Therefore, we transform problem (5) into a bilevel programming problem, with the upper-level variables being $C_{0,k}$ as follows:

$$\min_{\{C_{0,k}\}} \left(\begin{array}{l} \psi\left(C_{0,k}\right) = \min\limits_{\mathbf{p}_0, \{\mathbf{p}_k, \theta_k\}} \|\mathbf{p}_0\|^2 + \sum\limits_{k=1}^{K} \|\mathbf{p}_k\|^2 \\ \text{subject to (5b), (5c), (5e), (5f), (5g)} \end{array} \right), \tag{6a}$$

where $\psi\left(C_{0,k}\right)$ corresponds to the inner optimization problem with respect to variables $\mathbf{p}_0, \{\mathbf{p}_k, \theta_k\}$ and $\min_{\{C_{0,k}\}}\left(\psi\left(C_{0,k}\right)\right)$ is the outer optimization problem with respect to variables $\left\{C_{0,k}\right\}$ subject to the constraint $C_{0,k} \geq 0$, $\forall k$.

The general idea is to iteratively optimize the outer and inner optimization problems, i.e. the upper-level variables $C_{0,k}$, obtained by a PSO-based method [20], are the input parameters to solve the inner optimization problem $\psi\left(C_{0,k}\right)$. After that, based on the previous solution of the inner optimization function, the PSO algorithm updates the variables $C_{0,k}$, which are again used to solve the inner optimization problem, repeating the process until convergence. In more details, to find the approximately optimal values of the common rates, $C^*_{0,k}$, in the outer optimization problem, we propose a PSO-based method, described in the Sect. 3.2, that provides low computational complexity, high accuracy, and a high convergence speed. Second, for any given $C_{0,k}$, the inner optimization problem of (6) can be simplified into the following problem:

$$\min_{\mathbf{p}_0,\{\mathbf{p}_k,\theta_k\}} \quad \|\mathbf{p}_0\|^2 + \sum_{k=1}^{K}\|\mathbf{p}_k\|^2 \quad \text{subject to} \tag{7a}$$

$$\frac{\left|\mathbf{h}_k^H\mathbf{p}_k\right|^2}{\sum_{i=1,i\neq k}^{K}\left|\mathbf{h}_k^H\mathbf{p}_i\right|^2+\frac{\delta_k^2}{\theta_k}} \geq \phi_k, \quad \forall k \tag{7b}$$

$$\frac{\left|\mathbf{h}_k^H\mathbf{p}_0\right|^2}{\sum_{i=1}^{K}\left|\mathbf{h}_k^H\mathbf{p}_i\right|^2+\frac{\delta_k^2}{\theta_k}} \geq \varsigma, \quad \forall k, \tag{7c}$$

$$(5e), (5e), (5e),$$

where $\phi_k = \max\{2^{\gamma_k - C_{0,k}} - 1, 0\}$ and $\varsigma = 2^{\sum_{i=1}^{K} C_{0,i}} - 1$. In the following Sects. 3.3 and 3.4, we propose to solve the inner optimization problem (7) with two approaches: the SDR technique and an SCA-based iterative algorithm.

3.2 Particle swarm optimization algorithm to solve outer optimization problem (6)

We propose a PSO-based method [20] to obtain the approximately optimal k-th user's portion of the common rates, $\{C_{0,k}\}$. Let $I_{\max}$ and S, respectively, denote the maximum number of iterations and the number of particles in a swarm. Then, each particle's position represents a vector of K elements, $\{C_{0,1}, \ldots, C_{0,K}\}$. To guarantee constraint (5c), we need to limit the maximum value of $C_{0,k}$. Subsequently, we create a maximum value for $C_{0,k}$, denoted by $C_{0,k\max}$, which defines the search range of $C_{0,k}$ as $[0, C_{0,k\max}]$. Let $\mathbf{x}_n$, $\mathbf{v}_n$, and $\mathbf{p}^n_{best}$ denote the position, velocity, and individual best position of particle n, respectively. In addition, the global best position for all particles in the swarm is denoted by $\mathbf{g}_{best}$. The fitness function to minimize in PSO is denoted as $f(\mathbf{x}_n)$. The fitness function corresponds to the objective function obtained by solving problem (7) when the k-th user's portion of the common rates is $\mathbf{x}_n = \{C_{0,1_n}, \ldots, C_{0,K_n}\}$. As we mentioned earlier, we propose to solve problem (7) with two approaches: SDR technique and SCA-based iterative algorithm. Following [20], we use inertia weight parameter w for a velocity update and the acceleration coefficients c_1 and c_2.

The value of $C_{0,k\max}$ is defined by constraint (5c) in original problem (5) and by the values of $\phi_k = \max\{2^{\gamma_k - C_{0,k}} - 1, 0\}$ and $\varsigma = 2^{\sum_{i=1}^{K} C_{0,i}} - 1$ in problem (7). First, based on constraint (5c) and maximum available power constraint (5f), we define the limit for $C_{0,k\max}$ as follows:

$$C_{0,k\max} = \min_{1\leq i\leq K}\left\{\log_2\left(1+\frac{\left|\mathbf{h}_i^H\mathbf{p}_{0\max}\right|^2}{\delta_i^2}\right)\right\} \tag{8a}$$

$$C_{0,k\max} \leq \min_{1\leq i\leq K}\left\{\log_2\left(1+\frac{\|h_i\|^2 P_{\max}}{\delta_i^2}\right)\right\} \tag{8b}$$

Next, based on the values of $\phi_k = \max\{2^{\gamma_k - C_{0,k}} - 1, 0\}$ and $\varsigma = 2^{\sum_{i=1}^{K} C_{0,i}} - 1$, we can define the cases: $C_{0,k} < \gamma_k$ and $C_{0,k} \geq \gamma_k$.

In the first case, $C_{0,k} < \gamma_k$, we have a value for ϕ_k greater than zero, and the range of values for $C_{0,k}$ is $[0, \gamma_k)$.

In the second case, $C_{0,k} \geq \gamma_k$, the value of ϕ_k is zero, and constraint (7b) is always satisfied for any value of variables $\mathbf{p}_0, \{\mathbf{p}_k, \theta_k\}$. Furthermore, since the objective is to minimize the transmit power, we can decrease the value of $\left|\mathbf{h}_k^H\mathbf{p}_0\right|^2$ by reducing the value of ς, which minimizes the total transmit power. Therefore, in the second case, the optimal value of $C_{0,k}$ is γ_k. By combining the first and second cases, the maximum value of $C_{0,k}$ is $C_{0,k\max} = \gamma_k$.

Finally, by joining the two possible values for $C_{0,k\max}$, we have the following:

$$C_{0,k\max} = \min\left\{\gamma_k, \min_{1\leq i\leq K}\log_2\left(1+\frac{\|h_i\|^2 P_{\max}}{\delta_i^2}\right)\right\}, \quad \forall k. \tag{9}$$

We summarize the proposed PSO-based algorithm for problem (5) in Table 1.

3.3 First approach to solving inner optimization problem (7) using the SDR technique

In this subsection, we derive an optimal solution to problem (7) via SDR [21]. Define $\mathbf{P}_0 = \mathbf{p}_0\mathbf{p}_0^H$, $\mathbf{P}_k = \mathbf{p}_k\mathbf{p}_k^H$, and $\mathbf{H}_k = \mathbf{h}_k\mathbf{h}_k^H$, $\forall k$. Based on the properties $\|\mathbf{x}\|^2 = \mathbf{x}^H\mathbf{x}$, $\mathrm{Tr}(\mathbf{AB}) = \mathrm{Tr}(\mathbf{BA})$, and $x = \mathrm{Tr}(x)$, we are able to derive the following expressions: $\|\mathbf{p}_k\|^2 = \mathrm{Tr}(\mathbf{P}_k)$ and $\left|\mathbf{h}_k^H\mathbf{p}_i\right|^2 = \mathrm{Tr}(\mathbf{H}_k\mathbf{P}_i)$. Furthermore, the matrix variable, $\mathbf{P}_k = \mathbf{p}_k\mathbf{p}_k^H$, is equivalent to $\mathbf{P}_k$ being a rank-1 symmetric positive semidefinite (PSD) matrix, i.e. $\mathbf{P}_k \succeq 0$ and $\mathrm{rank}(\mathbf{P}_k) = 1$. Therefore, we can express problem (7) in the following equivalent form:

$$\min_{\mathbf{P}_0,\{\mathbf{P}_k,\theta_k\}} \quad \mathrm{Tr}(\mathbf{P}_0) + \sum_{k=1}^{K}\mathrm{Tr}(\mathbf{P}_k) \quad \text{subject to} \tag{10a}$$

Table 1 The proposed PSO algorithm based on problem (6) to solve problem (5)

1:	**inputs:** $I_{\max}$, S, w, c_1, c_2, $C_{0,k\max}$, $v_{\max}$, and variables $\{\mathbf{x}_n\}$, $n = 1, \ldots, S$.
2:	Set the iteration index $i = 1$
3:	Initialize population of particles having the positions $\mathbf{x}_n = \{C_{0,1_n}, \ldots, C_{0,K_n}\}$, which are randomly selected in $[0, C_{0,k\max}]$, and evaluate $f(\mathbf{x}_n)$ by solving inner optimization problem (7) by SDR or SCA.
4:	Find the index of the best particle: $\mathbf{g}_{best} = \arg\min_{1\le n\le N} f(\mathbf{x}_n)$.
5:	Initialize $\mathbf{p}^n_{best}$ to its initial position: $\mathbf{p}^n_{best} = \mathbf{x}_n, \forall n$.
6:	Initialize the particle's velocity: $\mathbf{v}_n = 0, \forall n$.
7:	**repeat**
8:	**For** each particle $n = 1, \ldots, S$, **do**
9:	Select random numbers: $r_1^n, r_2^n \sim U(0,1)$.
10:	Update particle's velocity: $\mathbf{v}_n \leftarrow w\mathbf{v}_n + c_1 r_1^n(\mathbf{p}^n_{best} - \mathbf{x}_n) + c_2 r_2^n(\mathbf{g}_{best} - \mathbf{x}_n)$
11:	Limit each element of vector $\mathbf{v}_n$ in $[-v_{\max}, v_{\max}]$.
12:	Update position of the particles: $\mathbf{x}_n \leftarrow \mathbf{x}_n + \mathbf{v}_n$.
13:	Limit each element of vector $\mathbf{x}_n$ in $[0, C_{0,k\max}]$.
14:	Evaluate $f(\mathbf{x}_n)$ and calculate corresponding precoders $\mathbf{p}_0, \{\mathbf{p}_k\}$ and the power split variables $\{\theta_k\}$ by solving inner optimization problem (7) with SDR or SCA when the common rate variables are $\mathbf{x}_n$.
15:	Update particle's best position: **if** $f(\mathbf{x}_n) < f(\mathbf{p}^n_{best})$ **then** $\mathbf{p}^n_{best} \leftarrow \mathbf{x}_n$. **end if**
16:	Update the global best position of the swarm: **if** $f(\mathbf{x}_n) < f(\mathbf{g}_{best})$ **then** $\mathbf{g}_{best} \leftarrow \mathbf{x}_n$, $\mathbf{p}_0^*, \{\mathbf{p}_k^*\} \leftarrow \mathbf{p}_{0_n}, \{\mathbf{p}_k\}_n$, $\{\theta_k^*\} \leftarrow \{\theta_k\}_n$. **end if**
17:	**end for**
18:	Update: $i \leftarrow i + 1$.
19:	**until** termination criterion is met or $i > I_{\max}$
20:	**outputs:** Set $f(\mathbf{g}_{best})$ as the minimum value of problem (5) at the optimal common rates $\{C_{0,1}, \ldots, C_{0,K}\} = \mathbf{g}_{best}$, optimal precoders $\mathbf{p}_0^*, \{\mathbf{p}_k^*\}$, and optimal power split ratios $\{\theta_k^*\}$.

$$-\mathrm{Tr}(\mathbf{H}_k\mathbf{P}_k) + \sum_{i=1, i\neq k}^{K} \mathrm{Tr}(\mathbf{H}_k\mathbf{P}_i)\phi_k + \frac{\phi_k\delta_k^2}{\theta_k} \le 0, \quad \forall k \tag{10b}$$

$$\frac{-\mathrm{Tr}(\mathbf{H}_k\mathbf{P}_0)}{\varsigma} + \sum_{i=1}^{K} \mathrm{Tr}(\mathbf{H}_k\mathbf{P}_i) + \frac{\delta_k^2}{\theta_k} \le 0, \quad \forall k \tag{10c}$$

$$-\mathrm{Tr}(\mathbf{H}_k\mathbf{P}_0) - \sum_{i=1}^{K} \mathrm{Tr}(\mathbf{H}_k\mathbf{P}_i) + \frac{\alpha_k}{\eta_k(1-\theta_k)} \le 0, \quad \forall k \tag{10d}$$

$$\mathrm{Tr}(\mathbf{P}_0) + \sum_{i=1}^{K} \mathrm{Tr}(\mathbf{P}_i) \le P_{\max} \tag{10e}$$

$$\mathbf{P}_0, \mathbf{P}_k \succeq 0, \quad \forall k \tag{10f}$$

$$\mathrm{rank}(\mathbf{P}_0), \mathrm{rank}(\mathbf{P}_k) = 1, \quad \forall k \tag{10g}$$

$$0 < \theta_k < 1, \quad \forall k. \tag{10h}$$

Problem (10) is still non-convex because of constraints (10g), which are not affine functions. Thus, we can drop constraints (10g) to obtain the SDP problem, called (10)-SDR. Using the convex optimization toolbox CVX [22], we can achieve the optimal solution to problem (10)-SDR. Problem (10)-SDR is an SDP problem that has $K+1$ matrix variables of size NxN and $3K+1$ linear constraints for matrix variables. Therefore, the computational complexity for solving problem (10)-SDR is $\mathcal{O}\left(\sqrt{KN}(K^3N^6 + K^2N^2)\log\left(\frac{1}{\zeta}\right)\right)$ given solution accuracy $\zeta > 0$ [17]. Let $\mathbf{P}_0^*, \{\mathbf{P}_k^*\}$ and $\{\theta_k^*\}$ denote the optimal solution to problem (7). If $\mathbf{P}_0^*, \{\mathbf{P}_k^*\}$ satisfies $\mathrm{rank}(\mathbf{P}_0^*) = 1$ and $\mathrm{rank}(\mathbf{P}_k^*) = 1$, $\forall k$ then the optimal precoders $\mathbf{p}_0^*, \{\mathbf{p}_k^*\}$ correspond to the optimal solution to problem (7).

On the other hand, if the rank of $\mathbf{P}_0^*, \{\mathbf{P}_k^*\}$ is larger than 1, we can use the Gaussian randomization method [23, 24] to generate candidate precoding vectors and use them to construct an approximate solution to problem (7). First, we perform eigen-decomposition of optimal precoding matrices $\mathbf{P}_0^* = \mathbf{U}_0\mathbf{\Lambda}_0\mathbf{U}_0^H$ and $\mathbf{P}_k^* = \mathbf{U}_k\mathbf{\Lambda}_k\mathbf{U}_k^H$. Second, we create candidate precoders $\mathbf{p}_0 = \mathbf{U}_0\mathbf{\Lambda}_0^{1/2}\mathbf{v}_0$ and $\mathbf{p}_k = \mathbf{U}_k\mathbf{\Lambda}_k^{1/2}\mathbf{v}_k$, where the elements of vectors $\mathbf{v}_0$ and $\mathbf{v}_k$ follow a complex circularly symmetric Gaussian distribution with zero mean and unit variance, i.e. $\mathbf{v}_k \in \mathbb{C}^N \sim \mathcal{CN}(0, \mathbf{I})$. However, the candidate precoders obtained through the Gaussian randomization procedure are not guaranteed to be feasible solutions to problem (7), so in order to convert the candidate precoders into feasible solutions to problem (7), we define scalar factors z_0 and z_k as solutions to the following problem:

$$\min_{z_0,\{z_k\}} \; z_0\|\mathbf{p}_0\|^2 + \sum_{k=1}^{K} z_k\|\mathbf{p}_k\|^2 \quad \text{subject to} \tag{11a}$$

$$-z_k\left|\mathbf{h}_k^H\mathbf{p}_k\right|^2 + \sum_{i=1, i\neq k}^{K} z_i\phi_k\left|\mathbf{h}_k^H\mathbf{p}_i\right|^2 + \frac{\delta_k^2\phi_k}{\theta_k} \le 0, \quad \forall k \tag{11b}$$

$$-z_0\left|\mathbf{h}_k^H\mathbf{p}_0\right|^2+\sum_{i=1}^{K}z_i\varsigma\left|\mathbf{h}_k^H\mathbf{p}_i\right|^2+\frac{\varsigma\delta_k^2}{\theta_k}\leq 0,\quad \forall k \tag{11c}$$

$$\frac{\alpha_k}{\eta_k(1-\theta_k)}-z_0\left|\mathrm{h}_k^H\mathbf{p}_0\right|^2-\sum_{i=1}^{K}z_i\left|\mathrm{h}_k^H\mathbf{p}_i\right|^2\leq 0,\quad \forall k \tag{11d}$$

$$z_0\|\mathbf{p}_0\|^2+\sum_{i=1}^{K}z_i\|\mathbf{p}_i\|^2\leq P_{\max} \tag{11e}$$

$$z_0,z_k\geq 0,\quad \forall k. \tag{11f}$$

Problem (11) is a linear program, and easily solved with the CVX toolbox of Matlab [22]. The summary of the Gaussian randomization method to obtain the approximate solution to problem (7) is presented in Table 2.

The total computational complexity of the proposed algorithm depends on $I_{\max}$, S, and the computational complexity in solving problem (10)-SDR. Hence, the computational complexity of the proposed SDR-based technique is $\mathcal{O}\left(I_{\max}S\left(\sqrt{KN}\left(K^3N^6+K^2N^2\right)\log\left(\frac{1}{\zeta}\right)\right)\right)$.

3.4 Second approach to solving inner optimization problem (7) using the SCA-based iterative algorithm

In this subsection, the SCA method [25] is applied to achieve convex approximations of the concave part in constraints (7b), (7c), and (5e) of problem (7). Inspired by the solution to the minimum transmit power problem in Section IV in [26], let us define $\tilde{\mathbf{p}}_k$ as an initial feasible point for the precoder $\mathbf{p}_k$. Consequently, we can insert $\mathbf{p}_k=\tilde{\mathbf{p}}_k+\Delta\mathbf{p}_k$ into the expression $\left|\mathbf{h}_k^H\mathbf{p}_k\right|^2$ of (7b), (7c), and (5e) as follows:

$$\left|\mathbf{h}_k^H\mathbf{p}_k\right|^2=\mathbf{p}_k^H\mathbf{h}_k\mathbf{h}_k^H\mathbf{p}_k=(\tilde{\mathbf{p}}_k+\Delta\mathbf{p}_k)^H\mathbf{h}_k\mathbf{h}_k^H(\tilde{\mathbf{p}}_k+\Delta\mathbf{p}_k) \tag{12}$$

$$\left|\mathbf{h}_k^H\mathbf{p}_k\right|^2\geq\tilde{\mathbf{p}}_k^H\mathbf{h}_k\mathbf{h}_k^H\tilde{\mathbf{p}}_k+2\mathbb{R}\left\{\tilde{\mathbf{p}}_k^H\mathbf{h}_k\mathbf{h}_k^H\Delta\mathbf{p}_k\right\}, \tag{13}$$

where (13) is derived by dropping the quadratic form, expressed as $\Delta\mathbf{p}_k^H\left(\mathbf{h}_k\mathbf{h}_k^H\right)\Delta\mathbf{p}_k$. Similar to the aforementioned, we can insert $\mathbf{p}_0=\tilde{\mathbf{p}}_0+\Delta\mathbf{p}_0$ into $\left|\mathbf{h}_k^H\mathbf{p}_0\right|^2$ as follows:

$$\left|\mathbf{h}_k^H\mathbf{p}_0\right|^2\geq\tilde{\mathbf{p}}_0^H\mathbf{h}_k\mathbf{h}_k^H\tilde{\mathbf{p}}_0+2\mathbb{R}\left\{\tilde{\mathbf{p}}_0^H\mathbf{h}_k\mathbf{h}_k^H\Delta\mathbf{p}_0\right\}. \tag{14}$$

The drawback with the SCA algorithm is that we need a feasible initial point, which in general is difficult to define. Inspired by the feasible point pursuit SCA algorithm in [27], we consider slack variables s_m with $m=\{1,\ldots,3K+1\}$, two for each of the following constraints: (7b), (7c), (5e), and one for the constraint (5f). The function of the slack variables is to guarantee the feasibility of the modified problem. Furthermore, we include a slack penalty to force the slack variables to take a value as close as possible to zero.

Finally, by introducing the convex approximation of $\left|\mathbf{h}_k^H\mathbf{p}_k\right|^2$ and $\left|\mathbf{h}_k^H\mathbf{p}_0\right|^2$, the aforementioned slack variables s_m, and a slack penalty, we can reformulate problem (7) as follows:

$$\min_{\mathbf{p}_0,\{\mathbf{p}_k,\theta_k,s_m\}}\ \|\mathbf{p}_0\|^2+\sum_{k=1}^{K}\|\mathbf{p}_k\|^2+\beta\sum_{m=1}^{3K+1}s_m\quad\text{subject to} \tag{15a}$$

$$\sum_{i=1,i\neq k}^{K}\phi_k\left|\mathbf{h}_k^H\mathbf{p}_i\right|^2+\frac{\phi_k\delta_k^2}{\theta_k}-\tilde{\mathbf{p}}_k^H\mathbf{h}_k\mathbf{h}_k^H\tilde{\mathbf{p}}_k-2\mathbb{R}\left\{\tilde{\mathbf{p}}_k^H\mathbf{h}_k\mathbf{h}_k^H\Delta\mathbf{p}_k\right\}\leq s_k,\quad\forall k \tag{15b}$$

$$\sum_{i=1}^{K}\left|\mathbf{h}_k^H\mathbf{p}_i\right|^2+\frac{\delta_k^2}{\theta_k}-\frac{\tilde{\mathbf{p}}_0^H\mathbf{h}_k\mathbf{h}_k^H\tilde{\mathbf{p}}_0}{\varsigma}-\frac{2\mathbb{R}\left\{\tilde{\mathbf{p}}_0^H\mathbf{h}_k\mathbf{h}_k^H\Delta\mathbf{p}_0\right\}}{\varsigma}\leq s_{k+K},\quad\forall k \tag{15c}$$

Table 2 The proposed algorithm for generating an approximate solution to problem (7) based on Gaussian randomization

1:	**inputs:** Number of randomizations N_{rand}, $min\_val=P_{\max}$, optimal solution of the (10)-SDR problem $\mathbf{P}_0^*$, $\{\mathbf{P}_k^*\}$.
2:	Perform the eigen-decomposition of each optimal matrix: $\mathbf{P}_0^*=\mathbf{U}_0\mathbf{\Lambda}_0\mathbf{U}_o^H$ and $\mathbf{P}_k^*=\mathbf{U}_k\mathbf{\Lambda}_k\mathbf{U}_k^H$, $\forall k$
3:	**For** $i=1:N_{rand}$
4:	Create vectors $\mathbf{v}_0^i$ and $\mathbf{v}_k^i$ where their elements follow a complex circularly symmetric Gaussian distribution with zero mean and unit variance.
5:	Generate the candidate precoders: $\mathbf{p}_0^i=\mathbf{U}_0\mathbf{\Lambda}_0^{1/2}\mathbf{v}_0^i$ and $\mathbf{p}_k^i=\mathbf{U}_k\mathbf{\Lambda}_k^{1/2}\mathbf{v}_k^i$, $\forall k$
6:	Solve problem (11) to find scalar factors z_0^i, z_k^i, $\forall k$.
7:	Convert the candidate precoders into feasible solutions to problem (7). $\mathbf{p}_0^i=\sqrt{z_0^i}\mathbf{U}_0\mathbf{\Lambda}_0^{1/2}\mathbf{v}_0^i$ $\mathbf{p}_k^i=\sqrt{z_k^i}\mathbf{U}_k\mathbf{\Lambda}_k^{1/2}\mathbf{v}_k^i$, $\forall k$
8:	Define $optval_i=\left\Vert\mathbf{p}_0^i\right\Vert^2+\sum_{k=1}^{K}\left\Vert\mathbf{p}_k^i\right\Vert^2$.
9:	**if** $optval_i<min\_val$ **then** $min\_val=optval_i$, $\mathbf{p}_0^*=\mathbf{p}_0^i$, $\mathbf{p}_k^*=\mathbf{p}_k^i$, $\forall k$ **end if**
10:	**end for**
20:	**outputs:** $\mathbf{p}_0^*$, $\{\mathbf{p}_k^*\}$.

$$\frac{\alpha_k}{\eta_k(1-\theta_k)} - \tilde{\mathbf{p}}_0^H \mathbf{h}_k \mathbf{h}_k^H \tilde{\mathbf{p}}_0 - 2\mathbb{R}\{\tilde{\mathbf{p}}_0^H \mathbf{h}_k \mathbf{h}_k^H \Delta\mathbf{p}_0\} + \sum_{i=1}^{K}\left(-\tilde{\mathbf{p}}_i^H \mathbf{h}_k \mathbf{h}_k^H \tilde{\mathbf{p}}_i\right) + \sum_{i=1}^{K}\left(-2R\{\tilde{\mathbf{p}}_i^H \mathbf{h}_k \mathbf{h}_k^H \Delta\mathbf{p}_i\}\right) \le s_{k+2K}, \quad \forall k \tag{15d}$$

$$\|\mathbf{p}_0\|^2 + \sum_{i=1}^{K}\|\mathbf{p}_i\|^2 - P_{\max} \le s_{3K+1} \tag{15e}$$

$$\Delta\mathbf{p}_0 = \mathbf{p}_0 - \tilde{\mathbf{p}}_0, \Delta\mathbf{p}_k = \mathbf{p}_k - \tilde{\mathbf{p}}_k, \quad \forall k. \tag{15f}$$

$$0 < \theta_k < 1, \quad \forall k. \tag{15g}$$

$$s_m \ge 0, \quad \forall m, \tag{15h}$$

where β is a factor to trade off the original objective function and the slack penalty. Note that problem 15 is always feasible, and variables $\mathbf{p}_0, \{\mathbf{p}_k, \theta_k\}$ are the optimal solutions to problem (7) if $s_m^* = 0$. In this way, we use factor $\beta >> 1$ to move the values of the slack variables towards zero, which permits us to obtain a feasible solution to problem (7).

Problem (15) is convex and can be directly solved by the convex optimization toolbox CVX [22]. We define $\mathbf{p}_{k,i}^*$ and $\mathbf{p}_{0,i}^*$ as the optimal solutions at the i-th iteration. Then, in each iteration, we update optimal solutions $\mathbf{p}_{k,i+1}^*$ and $\mathbf{p}_{0,i+1}^*$ as $\mathbf{p}_{k,i+1}^* = \mathbf{p}_{k,i}^*$ and $\mathbf{p}_{0,i+1}^* = \mathbf{p}_{0,i}^*$ until the algorithm converges. Problem (15) has $k+1$ vector variables of size N, and $3K+1$ linear constraints for vector variables. Hence, the computational complexity for solving problem (15) is $\mathcal{O}\left([N+3K+1]^{3.5}\right)$ [27]. Therefore, the total computational complexity of the proposed algorithm based on PSO with SCA is $\mathcal{O}\left(I_{\max}S[N+3K+1]^{3.5}\right)$.

We consider the initialization procedure for the WMMSE algorithm defined in Section V.A of [3] to define initial precoders $\mathbf{p}_{0,i}$ and $\mathbf{p}_{k,i}$. Note that the initialization procedure in [3] by itself does not guarantee feasible precoders for problem (7), so we must use the strategy of the slack variables described in problem (15) to overcome this infeasibility. The proposed algorithm to solve problem (7) is described in Table 3.

3.5 Baseline scheme to solve the minimum transmit power problem using SDMA

In this subsection, we describe the baseline scheme: space-division multiple access to solve the minimum transmit power problem in a MISO SWIPT system. In SDMA, the message m_k, intended for user k, is encoded into independent data streams, w_k. The precoding vectors, $\mathbf{q}_k \in C^{Nx1}$, map the symbols to the transmit antennas. Then, the SINR at user k is as follows:

$$\mathrm{SINR}_{k,SDMA} = \frac{\theta_k\left|\mathbf{h}_k^H \mathbf{q}_k\right|^2}{\theta_k \sum_{i=1, i\neq k}^{K}\left|\mathbf{h}_k^H \mathbf{q}_i\right|^2 + \delta_k^2}, \quad \forall k. \tag{16}$$

The instantaneous achievable rate at user k for streams w_k is denoted by $R_{k,SDMA} = \log_2\left(1+\mathrm{SINR}_{k,SDMA}\right)$. Moreover, the power harvested by the energy harvesting module of user k is given by

$$E_{k,SDMA} = \eta_k(1-\theta_k)\left(\sum_{i=1}^{K}\left|\mathbf{h}_k^H \mathbf{q}_i\right|^2\right), \quad \forall k. \tag{17}$$

In a similar way to the problem (5), our objective is to minimize the transmit power at the BS with the constraints of minimum information data rate and minimum EH. The design problem is formulated as follows:

$$\min_{\{\mathbf{q}_k, \theta_k\}} \sum_{k=1}^{K}\|\mathbf{q}_k\|^2 \quad \text{subject to} \tag{18a}$$

$$R_{k,SDMA} \ge \gamma_k, \quad \forall k \tag{18b}$$

$$\mathrm{E}_{\mathrm{k,SDMA}} \ge \alpha_{\mathrm{k}}, \quad \forall k \tag{18c}$$

$$0 < \theta_k < 1, \quad \forall k. \tag{18d}$$

Note that problem (18) can be reformulated based on SINR and solved in an optimal way with the SDR

Table 3 The proposed SCA algorithm based on problem (15) to solve problem (7)

1:	**inputs:** channel vectors $\mathbf{h}_k$, noise variances δ_k^2, minimum total rate of user-k γ_k, maximum transmit power $P_{\max}$, common rates $C_{0,k}$, minimum EH at the users α_k, energy harvesting efficiencies η_k, tolerance $\varepsilon > 0$, maximum number of iterations $Q_{\max}$, and factor β.
2:	Set the initial number $i = 0$ and initialize $\mathbf{p}_{0,i}$ and $\mathbf{p}_{k,i}$.
3:	Evaluate $ObjF_i = \|\mathbf{p}_{0,i}\|^2 + \sum_{k=1}^{K}\|\mathbf{p}_{k,i}\|^2$
4:	**repeat**
5:	Solve problem (15) with $\tilde{\mathbf{p}}_0 = \mathbf{p}_{0,i}$ and $\tilde{\mathbf{p}}_k = \mathbf{p}_{k,i}$. Denote solution to $\mathbf{p}_0, \mathbf{p}_k, \theta_k$ as $\mathbf{p}_0^*, \mathbf{p}_k^*, \theta_k^*$.
6:	Set $\mathbf{p}_{0,i+1} = \mathbf{p}_0^*, \quad \mathbf{p}_{k,i+1} = \mathbf{p}_k^*$ and update the iteration number $i \leftarrow i+1$.
7:	Evaluate $ObjF_i = \|\mathbf{p}_{0,i}\|^2 + \sum_{k=1}^{K}\|\mathbf{p}_{k,i}\|^2$
8:	**until** $\frac{\|ObjF_{i-1} - ObjF_i\|}{ObjF_{i-1}} < \varepsilon$ or $i \ge Q_{\max}$
9:	Set the obtained values as the optimal solutions: $\mathbf{p}_0^* \leftarrow \mathbf{p}_0, \mathbf{p}_k^* \leftarrow \mathbf{p}_k, \theta_k^* \leftarrow \theta_k$
10:	**outputs:** $\mathbf{p}_0^*, \{\mathbf{p}_k^*, \theta_k^*\}$.

technique following the same procedure as solution (P11) in Section IV of [10].

4 Simulation results

In this section, we present the simulation results to evaluate the performance of our proposed solutions in a MISO SWIPT RSMA system in comparison with an SDMA approach. In the simulations, we set $K = 2, N = 4, \delta_k^2 = \delta^2 = -60\,\text{dBm}, \gamma_k = \gamma, \alpha_k = \alpha, P_{\max} = 40$ dBm, and $\eta_k = \eta = 1, \forall k$. In addition, we consider Rician fading to model the channel, assuming a value of 40 dB for the signal attenuation from the BS to users. Thus, the channel vector is expressed as

$$\mathbf{h}_k = \sqrt{\frac{K_R}{1+K_R}}\mathbf{h}_k^{LOS} + \sqrt{\frac{1}{1+K_R}}\mathbf{h}_k^{NLOS}, \tag{19}$$

where $K_R = 5$ dB is the Rician factor, $\mathbf{h}_k^{LOS}$ follows the line-of-sight (LOS) deterministic component, and $\mathbf{h}_k^{NLOS}$ is the Rayleigh fading component, where each element is modeled as a circularly symmetric complex Gaussian random variable with zero mean and covariance of -40 dB. We apply the far-field antenna array model [28] to model the LOS component as follows:

$$\mathbf{h}_k^{LOS} = 10^{-4}\left[1 e^{-j\pi\sin(\phi_k)} e^{-j2\pi\sin(\phi_k)} \ldots e^{-j(N-1)\pi\sin(\phi_k)}\right]^T, \tag{20}$$

where $\phi_1 = -25^o$ and $\phi_2 = -60^o$ are the angles of direction from the BS to the two users considered in the simulations, and the distance between successive antenna elements at the BS is established as half of the carrier wavelength.

First, we investigated the results for problem (5) by using SDR- and the SCA-based algorithms. Furthermore, we compared the results with the SDMA scheme and the equal power splitting (EPS) ratio scheme, which consist of equal splits of the received power for the ID and EH modules, i.e., $\theta_1 = \theta_2 = 0.5$.

The parameters used for the PSO algorithm were as follows: the number of particles in the swarm, $S = 10$, acceleration coefficients $c_1 = 1.494$ and $c_2 = 1.494$, and inertia weight $w = 0.7$. The maximum number of iterations was obtained from Fig. 3, where we present the convergence behavior of the objective function versus the iteration index for an average of several channel realizations with the value of the minimum EH equal to -15 dBm and minimum rates equal to 5, 5.1, 5.2, and 5.3 bit/s/Hz. We can see that the objective function quickly converges to a stable value from iteration index 12. Therefore, we set the maximum number of iterations to $I_{\max} = 15$. Furthermore, note that the difference between the transmit power on the first iteration and the transmit power on the 15th iteration is less than 0.01%, which allows us to conclude that a good enough approximation to the optimal result can be achieved even using less than 15 iterations.

Figure 4 shows the performance of a various number of randomizations, N_{rand}, for the SDR-based technique by using minimum EH equal to -21 dBm and minimum rates equal to 6, 5.9998, and 5.9995 bit/s/Hz. We can see that after 15 randomizations, we achieve a stable condition, and then, we select the number of randomizations equal to 15. It is noteworthy that the transmit power for the first randomization has about a 0.01% difference with respect to the transmit power at the 15th iteration, which permits reducing the number of randomizations if we want to decrease the computational complexity.

Figure 5 shows the convergence behavior of the SCA-based algorithm by using minimum EH equal to -21 dBm and minimum rates equal to 6, 3, and 1 bit/s/Hz. We can see that after 10 iterations, we can achieve stable performance; however, for the first five iterations, we notice a drastic change in the transmit power value. Furthermore, the value of the relative change of the actual objective value with respect to the value in the last iteration, i.e.

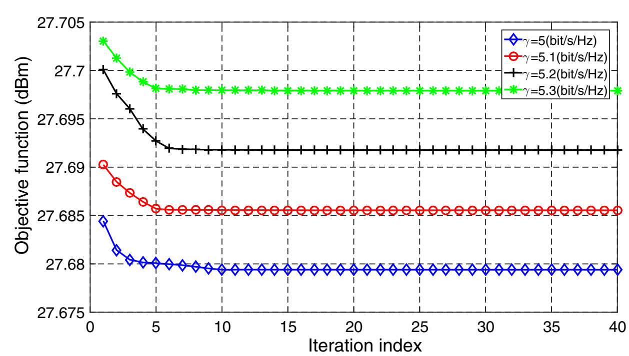

Fig. 3 The convergence behavior of the PSO-based algorithm with different minimum rates

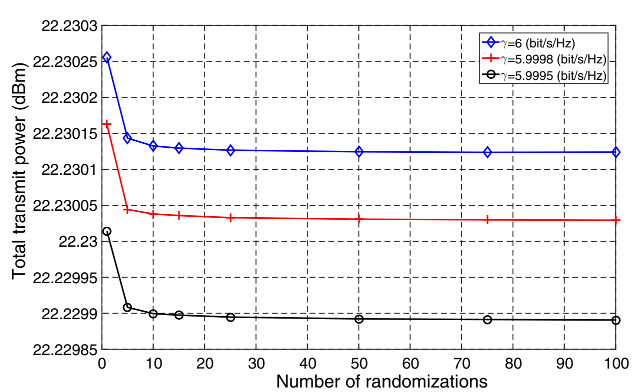

Fig. 4 Performance of various numbers from randomizations in SDR

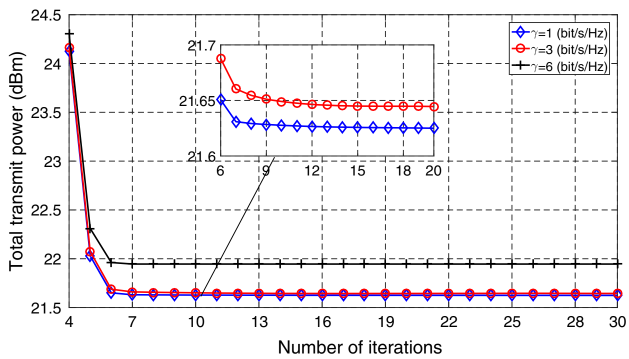

Fig. 5 Performance with various numbers of iterations in SCA

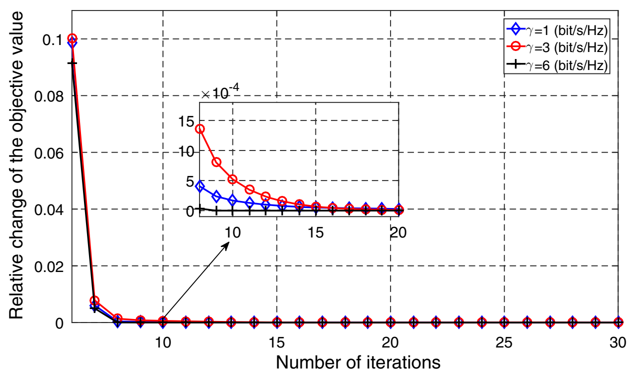

Fig. 6 Relative change of the objective value with various numbers of iterations in SCA

$\frac{|ObjF_{i-1}-ObjF_i|}{ObjF_{i-1}}$, is represented in Fig. 6, where again we observe remarkable changes during the first 10 iterations. Since the channel vectors are not the same, and each iteration produces considerable changes, we define the maximum number of iterations as $Q_{\max} = 20$, and tolerance $\varepsilon = 1e-4$.

By comparing the computational complexity of the SDR-based technique, $\mathcal{O}(\sqrt{KN}(K^3N^6 + K^2N^2))$, and the computational complexity of the SCA-based algorithm of $\mathcal{O}\left([N+3K+1]^{3.5}\right)$, we notice that for the number of antennas, $N = 4$, analyzed in previous simulations, the greater number of iterations needed in the SCA algorithm to achieve a tolerance of $\varepsilon = 1e-4$ makes the computational complexity in the SCA schemes superior, in comparison with the SDR approach.

Figure 7 presents the transmit power at the BS versus the minimum required rate by applying the PSO-based algorithm with the SDR- and SCA-based approaches to solve problem (5) and by using the SDMA and EPS schemes over several random channel realizations, with minimum EH equal to $\alpha = -15$ dBm. We see that the RS approach achieves a lower total transmit power in comparison with the SDMA and EPS schemes, because RS implements SIC

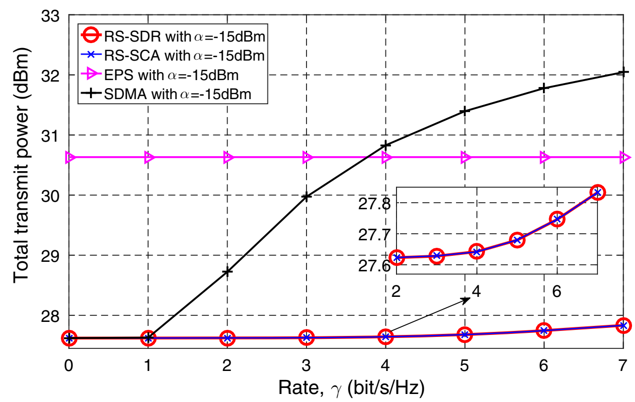

Fig. 7 Transmit power at the BS versus minimum rate required by the users

at the receiver, which permits user k to decode and remove the interference from the common message, increasing the SINR value at the receiver compared with the SDMA approach. Therefore, in the RS approach, through an optimal power allocation to the common and private messages and an optimal selection of the PS ratios, which can approximately be obtained with the proposed solution, the users can achieve a lower transmit power to satisfy the minimum rate constraints (5b) in comparison with the SDMA scheme. In the case of the EPS approach, the PS ratios are fixed at $\theta_k = 0.5$, which allows to easily obtain the rate constraints (5b) in our simulations, and makes the total power at the BS depends mostly in the minimum EH constraints (5e). Furthermore, the optimal values achieved by SDR and the SCA algorithm to solve problem (5) are the same, which confirms the results obtained for the RS approach.

Figure 8 shows the total transmit power at the BS versus the minimum harvested power, with minimum rates equal to $\gamma = 3$ bit/s/Hz and $\gamma = 6$ bit/s/Hz. We observe that the total transmit power increases with increasing values of the required EH because users require more energy from the BS to satisfy the EH constraint. We can see that the RS approach using SDR or SCA achieves a lower total transmit power in comparison with the SDMA and EPS schemes, particularly the RSMA method can reduce the total transmit power in around 2.4 dBm at $\gamma = 3$ bit/s/Hz and 4 dBm at $\gamma = 6$ bit/s/Hz compared with the SDMA scheme. This behavior is because the users, in the RS approach, remove the interference from the common signal via SIC at the ID module, which allows increasing the SINR at the ID module, reducing the required amount of the PS ratio variables (increasing the portion of the incoming energy signal that is sent to the EH module). Therefore, under the same value of the minimum required rate, γ_k, the RS approach needs a lower transmit power from the BS to satisfy the EH constraint (5e) in comparison

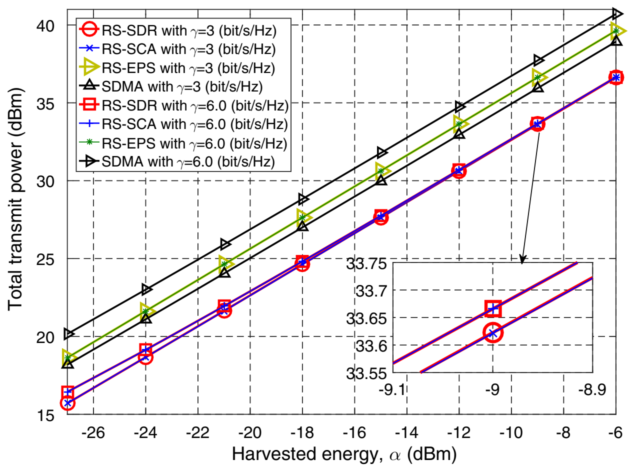

Fig. 8 Transmit power at the BS versus minimum EH required by the users

with the SDMA scheme. Furthermore, in the RS approach, we see that the total transmit power becomes more independent of the minimum rate constraint as we increase the EH requirement (e.g. from -18 to -6 dBm) because the SIC of the common message helps to easily satisfy the rate constraint (5b) and the EH constraint (5e) dominate the required value of transmit power from the BS. In the case of the EPS approach, the PS ratios $\theta_k = 0.5$ permit easily satisfy the minimum rate constraint (5b) in our numerical simulations, leading to a total transmit power that only depends on the EH requirement. Consequently, the EPS technique is not able to achieve a good balance between the ID and EH performance. In addition, the SDR and SCA schemes present the same results in the performed simulations, which substantiates the results obtained for the proposed system.

Figure 9 presents the total transmit power at the BS versus the number of antennas, where the minimum rate is

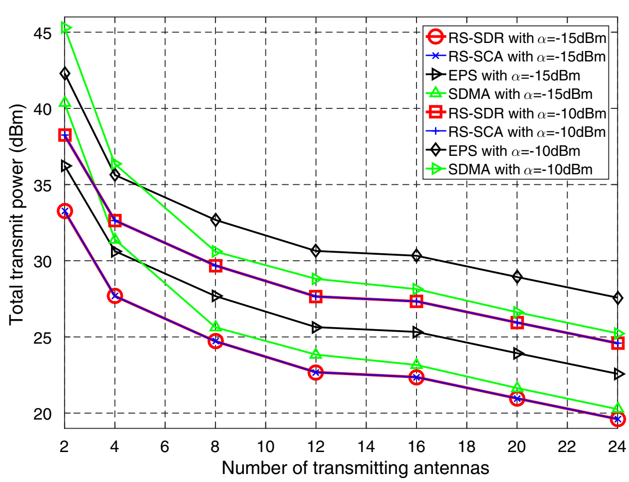

Fig. 9 Transmit power at the BS versus the number of transmitting antennas

$\gamma = 5$ bit/s/Hz, and we use minimum EH values equal to $\alpha = -15$ dBm and $\alpha = -10$ dBm. We observe that the total transmit power decreases with an increase in the number of antennas, because we can exploit the extra degrees of freedom to reduce the transmit power when the BS has more antennas. Moreover, the SDR- and the SCA-based algorithms achieve lower transmit power, compared with SDMA and EPS schemes. Like the previous Figs. 8 and 9, we attribute this behavior of the RS approach to the SIC performed at the ID module of the users and the capacity of the proposed solution to obtain an approximate optimal PS ratios selection and power allocation of the common and private messages, which improves the SINR and EH obtained at the users.

5 Conclusion

We propose a multi-user SWIPT MISO system using the RSMA method. We investigate the minimization problem of the total transmit power at the BS under the constraints of a minimum rate for users, minimum EH by users, and maximum power available at the BS. The PSO-based algorithm combined with the SDR- or SCA-based approaches were proposed to solve the problem and obtain the approximate optimal solutions. Simulation results show that the proposed PSO-based algorithm can achieve a fast convergence within 15 iterations and the proposed SDR- and SCA- based approaches achieve the same total transmit power at the BS, which confirms the results obtained for the RSMA approach. In addition, we have found that the computational complexity of the SDR-based technique is lower in comparison with that in the SCA-based technique under the number of antennas and parameters used in the proposed scenarios. Finally, numerical simulations have shown that proposed RSMA scheme outperforms the state-of-art SDMA scheme, particularly we refer to the scenario varying the minimum EH requirement at the users, where the RSMA method can reduce in around 3 dBm the total transmit power at the BS compared with the SDMA scheme.

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