A Solution for Building Motion Tracking System with Model Predictive Control for Autonomous Vehicles

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***Abstract* - With linearization solution for the model of wheels and vehicle according to model predictive control (MPC), in which vehicle models are linearized by a sets of steering angles with the assumption that these steering angles can help the vehicle to move to the last destination in a steady state with the model predictive control. In addition, the model's control input uses the force factor of the front wheel and the equivalent cornering wheel stiffness linearizes that of the rear wheel. This paper proposes a solution for building motion tracking system based on linear model for autonomous vehicles, aiming at minimizing the deflection movement at high velocity. The performance of the proposed solution is evaluated through simulation, then orientations of applied research are suggested for practical autonomous vehicle problems.**

Keywords— Model Predictive Control, Autonomous vehicle, motion planning, path planning, intelligent transportation systems.

I. INTRODUCTION

In the development of autonomous vehicles technology, in addition to the concerned factors such as the purpose of enhancing driving safety, reducing traffic congestion, vehicle exhaust problems, improving energy efficiency, the problem of navigational control for vehicles with the desire to drive the vehicle accurately and stably also plays a significant role in controlling the movement of the vehicle. Hence, many studies have been carried out during the past years such as the use of nonlinear vehicle and wheels models to simulate vehicle feedback when traveling at high velocities with large steering angles [3]. However, the simulation in this form is difficult to achieve optimal computational efficiency [1], or the building of the vehicle model assuming small steering angle and wheel model with linear ratio was studied to track the motion vehicle [2,6], it is still difficult if the sideslip angle and steering angle are more than 5^o, this model will not work correctly. In the study [4], the authors present a linear method to deal with the wheel in the MPC, but this method still does not achieve accuracy when predictive horizon is big. Therefore adjusting the parameters for a certain predictive model to actual values could be a task that needs to be studied and is full of challenges.

In this paper, with the addition of some assumptions, we propose a method to allow linear vehicle model and non-linear wheels according to the MPC scheme. Since then, we offer solutions for designing motion tracking controller using the model predictive control, which uses the deviation between the direction of the vehicle's motion and the vehicle velocity vector as the control reference state.

The next section of the paper introduces the MPC scheme's brief information and the linear method for these models, thereby we propose solutions for building a motion tracking system using the MPC approach. The last section is an experimental simulation to confirm the approach and suggests further research orientations for autonomous vehicle problem.

II. BUILDING MODEL FOR THE PREDICTIVE CONTROLLER

Before building a linear method for vehicle models using predictive data, we present the model predictive control system performed by the following differential equation:

$$x^* = f(x, u); \quad x \in X, u \in U \qquad (1)$$

Where $X \subset R^n$ is a convex and closed set, $U \subset R^m$ is a convex and compact set, x is the current state, x^* is the future state and u is the input control of the system.

At the same time, $x(k)$ and $u(k)$ are used to control action and denote the status at the time of sampling k.

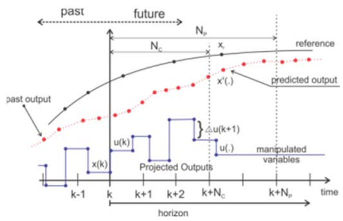

Figure 1. Description the principle of MPC

In the MPC scheme, the planning model $f(.)$ predicts the future states of the system and the control aim is to control $x^u(.)$ of the state trajectory to x_r of the desired state on a limited predictive N_P by applying the control order $u(.)$.

The cost functions $J_{N_c N_p}(.,.)$ are defined as follows:

$$J_{N_c N_p}(x^u(.), \triangle u(.)) = \sum_{i=0}^{N_{p-1}} \left(x(k+1) - x_r(k+i)\right)^T Q\left(x(k+1) - x_r(k+i)\right) + \sum_{i=0}^{N_{c-1}} \triangle u(k+i)^T R \triangle u(k+i) \qquad (2)$$

Where $\triangle u(.) = \{\triangle u(k), \triangle u(k+1), \dots, \triangle u(k+N_c-1)\}$ is the incremental sequence of input signal $x^u(.) = \{x(k+1), x(k+2, \dots, x(k+N_P)\}$, N_C is the control horizon having constraint $N_C \le N_P$, and Q, R are weighting matrices.

At the time of sampling k, MPC solves the optimal problem as follows:

$$\min_{\triangle u(.), x^u(.)} J_{N_C, N_P}(x(k), u(k-1), \triangle u(.)) \qquad (3)$$

Where

$x(k+i+1) = f(x(k+i), u(k+i))$ with $i = 0, \dots, N_P - 1$

$u(k+i) = u(k-1) + \sum_{j=0}^{i} \triangle u(k+j)$ with $i = 0, \dots, N_C - 1$

$\triangle u(k+i) \in \triangle U$ with $i = 0, \dots, N_C - 1$

$u(k+i) \in U$ with $i = 0, \dots, N_C - 1$

$x(k+i) \in X$ with $i = 1, \dots, N_P$

The optimal solution is affirmed to yield by $\triangle u^*(.)$ and therefore, the input control is defined as follows:

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$$u(k) = u(k-1) + \Delta u^*(k) \tag{4}$$

Resolving the optimal problem of equation (3) is a work that requires computation. Furthermore, the system's order, the nonlinearity in the planning model $f(.)$ and the length of the control horizon N_C are the main factors that require high levels of computation [5]. Normally, the introduction of the linear-quadratic optimization problem is an effective solution to improve computational efficiency.

II.1 Building nonlinear models

For the dynamic model of vehicle (Figure 2), it is represented by a single line with three position states and two velocity states. In order to implement the optimal problem of convex δ, the longitudinal velocity U_x is not allowed to change, although the external controller can be used to track the desired velocity configuration. Assuming the velocity in predictive horizon is fixed when planning this process model.

The yaw slip r, and the sideslip angle β are computed by the equation of motion, as follows [10]:

$$\dot{\beta} = \frac{F_f\cos(\delta)+F_r}{mU_x} - r \tag{5}$$

$$\dot{r} = \frac{l_f F_f \cos(\delta) - l_r F_r}{I_{zz}} \tag{6}$$

Where m is the vehicle mass, I_{zz} is the yaw inertia, F_r and F_f are the lateral forces of the rear and the front axles, l_r and l_f are the lengths from the center to the rear and the front axles.

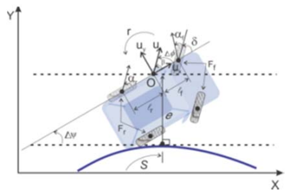

Figure 2. Dynamic model for autonomous vehicle

In the wheel model, the side force of the wheel is modeled using the Fiala brush wheel model [9], as follows:

$$F_{[f,r]} = f_{wheel}(\alpha, F_z) \tag{7}$$

Where:

$$f_{wheel}(\alpha, F_z) = \begin{cases} -\mu F_z sgn(\alpha) \;\; if \;\; |\alpha| > arctan\left(\frac{3\mu F_z}{C_\alpha}\right) \\ else \; - C_\alpha \tan(\alpha) + \frac{C_\alpha^2}{3\mu F_z}|\tan(\alpha)|\tan(\alpha) - \frac{C_\alpha^3}{27\mu^2 {F_z}^2} tan^3(\alpha) \end{cases} \tag{8}$$

F_z is the normal force, μ is the friction coefficient, and C_α is the cornering stiffness (this stiffness is usually evaluated at the slip angle for wheel equal 0).

The slip angles at front wheel α_f and the rear α_r use small-angle approximation, computed as follows:

$$\alpha_f = \beta + \frac{l_f.r}{U_x} - \delta \tag{9}$$

$$\alpha_r = \beta - \frac{l_r.r}{U_x} \tag{10}$$

Where δ is the steering angle.

The front lateral force F_f with the desire to create optimizing the MPC, then map to the front steering angle δ, as follows:

$$\delta = \beta + \frac{l_f.r}{U_x} - f_{wheel}^{-1}(F_f) \tag{11}$$

The motion tracking model (Figure 2) and the vehicle's relative position with desired path may be decided by distance s along the path, deviation in motion direction $\Delta\psi$ and lateral deviation e. Thus, the performing model is formed as follows:

$$\Delta\psi = r - U_x \kappa(s) \tag{12}$$

$$\dot{s} = U_x\cos(\Delta\psi) - U_y\sin(\Delta\psi) \tag{13}$$

$$\dot{e} = U_x\sin(\Delta\psi) + U_y\cos(\Delta\psi) \tag{14}$$

Where $\kappa(s)$ is the curvature at s of the desired path.

II.2 Building linear models

1) The most common linear method is to assume a small angle (when $\delta < 5^0$ and $\cos(\delta) \approx 1$), then the nonlinear dynamic model of the vehicle is written as follows:

$$\dot{\beta} \approx \frac{F_f + F_r}{mU_x} - r \tag{15}$$

$$\dot{r} \approx \frac{l_f.F_f - l_r.F_r}{I_{zz}} \tag{16}$$

Consider the assumptions below:

Assumption 1: Supposing increasing the steering angle is fixed at all steps on each predictive horizon, and at the same time it is independent of the control sequence $u(.)$ ie:

$$\Delta\, \delta_a(k+i) = \Delta\delta_{N_P}, \text{ with } i = 0, \dots, N_{P-1} \tag{17}$$

Assumption 2: Assuming the vehicle will move to the desired destination at the step N_P of the predictive horizon, and then operate on the trajectory without deviation.

Therefore, at step N_P the vehicle's operation is assumed to be at steady state, so:

$$e_a(k+N_P) = 0 \tag{18}$$

$$\delta_a(k+N_P-1) = \delta_{ss}(k+N_P-1) \tag{19}$$

$$\alpha_{[f,r],a}(k+N_P-1) = \alpha_{[f,r],ss}(k+N_P-1) \tag{20}$$

Where: δ_a is the steering angle, $\alpha_{[f,r],a}$ is the slip angle, e_a is the lateral deviation, δ_{ss} is the steering angle at steady state and $\alpha_{[f,r],ss}$ corresponding slip angle.

In vehicle operating condition when entering corners at steady state, set the value $\dot{r} = 0$ in expression (13), at this time the force of the front and rear wheels is computed as follows:

$$F_f^{ss} = \frac{ml_r}{2L}U_x^2\kappa \tag{21}$$

$$F_r^{ss} = \frac{ml_f}{2L}U_x^2\kappa \tag{22}$$

Therefore, the steering angle at steady state relates to the problem of sideslip at front and rear wheels dynamically through the following expression:

$$\delta_{ss} = L\kappa - \alpha_{f,ss} + \alpha_{r,ss} \tag{23}$$

Where $L = l_f + l_r$ is the wheel base. $\alpha_{f,ss}$ and $\alpha_{r,ss}$ can be computed by the inverse wheel model $f_{wheel}^{-1}(f_{[f,r]})$.

According to the above assumptions, the assumption of increasing the steering angle can be computed as follows:

$$\Delta\delta_a(k+i) = \frac{\delta(k) - \delta_a(k+N_P-1)}{N_P} \text{ with } i = 0, \dots, N_P - 1 \tag{24}$$

Thus, it can be seen that when the vehicle's velocity changes, the driving angle increase is set as follows:

$$\Delta\delta_{N_P} = \begin{cases} \Delta\delta_a & if\ |\Delta\delta_a| \le \Delta\delta_{max} \\ else, & sign(\Delta\delta).\Delta\delta_{max} \end{cases} \quad (25)$$

and the assumed steering angle at each predicted horizon is:

$$\delta_a(k+i) = \delta(k) + i.\Delta\delta_{N_P} \text{ with } i = 0, \ldots, N_P - 1 \quad (26)$$

Combining (26) with (5) and (6) will have a new linear version for the nonlinear vehicle model as follows:

$$\dot{\beta}(k+i) = \frac{F_f(k+i)\cos(\delta_a(k+i)) + F_r(k+i)}{mU_x} - r(k+i) \quad (27)$$

$$\dot{r}(k+i) = \frac{l_f F_f(k+i).\cos(\delta_a(k+i)) - l_r F_r(k+i)}{I_{zz}} \quad (28)$$

with $i = 0, \ldots, N_P - 1$

For the wheel model, its control input is determined as the front lateral force, and the nonlinear momentum of the rear wheel force must be accurately computed to be sure to approximate the MPC with the optimization problem.

On the predictive horizon, the concerning stiffness in the linear model is computed as follows:

$$\bar{C}_r = \frac{(\bar{F}_{r,ss} - \bar{F}_r)}{\bar{\alpha}_{r,ss} - \bar{\alpha}_r} \quad (29)$$

therefore, the rear lateral force predicted to be computated by the approximate expression:

$$F_r(k+i) = \bar{F}_r - \bar{C}_r(\alpha_r(k+i) - \bar{\alpha}_r) \quad (30)$$

with $i = 0, \ldots, N_P - 1$. Where, $\alpha_r(k+i)$ can be computed by (10).

Thus, the motion equation is performed by the linear equations as follows:

$$\dot{r}(k+i) = \frac{l_f F_f(k+i)\cos(\delta_a(k+i)) - l_r\left[\bar{F}_r - \bar{C}_r(\beta(k+i) - \frac{l_r.r(k+i)}{U_x} - \bar{\alpha}_r\right]}{I_{zz}} \quad (31)$$

$$\dot{\beta}(k+i) = \frac{F_f(k+i)\cos(\delta_a(k+i)) + \left[\bar{F}_r - \bar{C}_r(\beta(k+i) - \frac{l_r.r(k+i)}{U_x} - \bar{\alpha}_r\right]}{mU_x} - r(k+i) \quad (32)$$

with $i = 0, \ldots, N_P - 1$

In the motion model, creating small angle approximations for β and $\Delta\psi$ is as follows:

$$\Delta\dot{\psi}(k+i) = r(k+i) - U_x\kappa(k+i) \quad (33)$$

$$\dot{e}(k+i) = U_x(\beta(k+i) + \Delta\psi(k+i)) \quad (34)$$

$$\dot{s}(k+i) = U_x \quad (35)$$

with $i = 0, \ldots, N_P - 1$

Since the assumption of velocity does not change on the predictive horizon, the longitudinal distance along the trajectory can take an axiom as follows:

$$s(k+i) = s(k) + \sum_{j=0}^{i} U_x \text{ with } i = 0, \ldots, N_P \quad (36)$$

III. BUILDING MOTION TRACKING SYSTEM BY MPC

The problem for this solution is to use the initial course deviation for motion direction $\Delta\psi$ which does not increase the vehicle's capacity to the maximum level as a control reference value. The deviation $\Delta\varphi$ is the angle between the direction of motion and the vehicle velocity vector, indicating the true deviation of the direction of the vehicle motion and also that of the lateral deviation (Figure 3). When $\Delta\psi$ is close to the value of $\Delta\varphi$ and the side slip β is small, the $\Delta\psi$ -based controller can keep track deviation within a not large range. Yet when the distinction between $\Delta\varphi$ and $\Delta\psi$ is not small, especially close to the processing limit which the rear slip angle is high and the difference rate is high, the side slip value β is high. The controller based on $\Delta\psi$ will not make the tracking deviation minimized. This is particularly significant and cannot be ignored when the vehicle moves through corners with a physical limit of wheel friction, in which the slope β of the vehicle can reach 5^0.

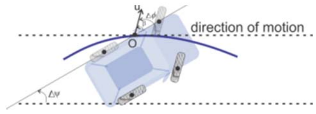

Figure 3. Relationship between deviation, sliding angle and direction of motion

According to the principles of vehicle dynamics, the deviation is computed as follows: $\Delta\varphi = \Delta\psi + \beta$ (37)

Because the lateral deviation is not in a steady state, the required value $\Delta\varphi$ must be zero and the value of $\Delta\psi$ is different from 0. Therefore, the reference state when building the motion controller should be the direct deviation $\Delta\varphi$.

From equations (31) - (36), we can get the discrete vehicle model by using the zero-order hold a discretization method as follows:

$$\mathrm{x(k+1)} = \mathrm{B_{F_f}F_f(k)} + \mathrm{A_c x(k)} + B_\kappa(k) + d_{\bar{\alpha}_r} \quad (38)$$

Applying integral effects to eliminate static gaps due to the uncertainty of the model, discrete models can be written in ascending form as follows:

$$\xi(k+1) = A\xi(k) + B_1\Delta F_f(k) + B_2\kappa + d \quad (39)$$

$$\eta(k) = C\xi(k) \quad (40)$$

$$F_f(k) = F_f(k-1) + \Delta F_f(k) \quad (41)$$

Where $\xi(k) = [x(k) \quad F_f(k-1)]^T$ is the expanded state vector, $\eta(k) = [\Delta f(k) \quad e(k)]^T$ is the output vector and $A = \begin{bmatrix} A_c & B_{F_f} \\ 0 & I \end{bmatrix}$, $B_1 = \begin{bmatrix} B_{F_f} \\ I \end{bmatrix}$, $B_2 = \begin{bmatrix} B_\kappa \\ 0 \end{bmatrix}$, $d = \begin{bmatrix} d_{\bar{\alpha}_r} \\ 0 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^T$

The final objective is to use the motion controller to bring to the optimal result of the front force input $F_f(k)$, so that the vehicle can maintain stability at the limit that it can handle and the lateral motion deviation can be minimized.

In the model building process, the building of constraints aiming at assuring the safety is determined by the limit of two important indicators about vehicle stability. According to the assumptions for the problem moving into a corner at steady state and the given wheel model, the limits of r and β reflect the wheel's maximum ability. The slip coefficient in the maximum steady state is expressed as the following:

$$r_{max} = \frac{g\mu}{U_x} \text{ , where } g \text{ is the vehicle's gravity} \quad (42)$$

With the slip coefficient r, the side slip β of the vehicle reaches the maximum level when the impact force on the rear wheel reaches the saturation level, so:

$$\beta_{ss,max} = \alpha_{r,sat} + \frac{l_r r}{U_x} \quad (43)$$

$$\alpha_{r,sat} = tan^{-1}\left(\frac{3mg\mu}{C_{\alpha_r}}\frac{l_f}{l_f+l_r}\right) \quad (44)$$

with $\alpha_{r,sat}$ is the saturation wheel angle and C_{α_r} is the rear concerning stiffness.

The above constraints can be expressed through the following inequality: $|H_v\xi(k)| \leq G_v$ (45)

where $H_v = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & -\frac{l_r}{U_x} & 0 & 0 & 0 \end{bmatrix}$, $G_v = [r_{max} \quad \alpha_{r,sat}]^T$

Thus, the optimal problem for model predictive control can be rewritten as follows:

$$\min_{\Delta F_f, \varepsilon_v} J_{N_P} = \sum_{i=1}^{N_P}(\eta(k+i))^T Q\eta(k+i) + \sum_{i=1}^{N_C} R\left(\Delta F_f(k+i)\right)^2 + W\varepsilon_v \quad (46)$$

$$= \sum_{i=1}^{N_P}\xi(k+i)^T C^T QC\xi(k+i) + \sum_{i=1}^{N_C} R\left(\Delta F_f(k+i)\right)^2 + W\varepsilon_v$$

Satisfy the conditions:

$H_v\xi(k+i) \leq G_v + \varepsilon_v$ with $\forall i$

$\left|\Delta F_f(k+i)\right| \leq \Delta F_{f,max}$ with $i = 0, \ldots, N_c - 1$

$\Delta F_f(k+i) = 0$ with $i = N_C, N_C+1, \ldots, N_P - 1$

$\left|F_f(k+i)\right| \leq F_{f,max}$ with $i = 0, \ldots, N_c - 1$

Where $\Delta F_f = [\Delta F_f(k), \Delta F_f(k+1), \ldots, \Delta F_f(k+N_c-1)]^T$ is a sequence of future input increases and Q, R and W are weighted matrix with appropriate dimensions. $\Delta F_{f,max}$ and $F_{f,max}$ are the ability to change velocity and maximum lateral force.

$|H_v\xi(k)| \leq G_v$ is built based on the assumptions of steady state. It is the vehicle status that can exceed the limit but can come back to the limit range after a short period of operation. Therefore, to make sure the optimal problem is always feasible, it is necessary to use the non-negative compensation variable ε_v. At this time the solution vector in the equation (46) is expanded as follows:

$$\Delta U^* = [\Delta F^*{}_f, \varepsilon^*{}_v]^T \quad (47)$$

Whether or not the optimal input of the front lateral force is achieved depends on the first value of the following optimal solution set: $F^*{}_f = F_f(k-1) + \Delta F^*{}_f(k)$ (48)

In addition, the steering angle δ will be applied by mapping $F^*{}_f(k)$ into the equation $\delta = \beta + \frac{l_f r}{U_x} - f_{wheel}^{-1}(F_f)$

IV. EXPERIMENTAL SIMULATION

In order to have the bases to evaluate our proposed solution, our group carries out experimental simulations the procedures in the environment of matlap. With the aim of ensuring the reliability and objectivity when evaluating, we conduct the simulation with 2 scenarios on 2 different path models (Figures 4a and 4b) designed with different curvature, road structure divided into 3 segments, in which the middle point of each segment is determined for comparison evaluation.

(a) **(b)**

Figure 4. Road structure in simulation

Applying the nonlinear model to the dynamic structure of vehicles (5) - (6), wheel models (8), motion-direction tracking model (12) - (14) to compute the vehicle state. In this problem, we apply Stanley method [11] to track the desired motion direction in both simulation scenarios with the model of constant velocity and the regulation of controlling steering angle is computed as follows:

$$\delta = \Delta\psi + tan^{-1}\left(\frac{k_p e}{U_x}\right), \quad k_p \text{ is gain parameter.} \quad (49)$$

The process of simulation is done with the sampling time $T_s = 0.02s$. At the same time, there needs to be a balance between control performance and computational complexity so the value of $N_c = 30$ and $N_P = 60$ are set, the weight matrices are adjusted through simulation and these values are determined as follows:

$$Q = \begin{bmatrix} 1500 & 0 \\ 0 & 5 \end{bmatrix}, R = 1, W = [10 \quad 10 \quad 10 \quad 10]$$

The corresponding transverse force U_x and the simulated steering angle R are set in scenario 1 as $U_x = 45km/h$ and $R = 30m$, in scenario 2 as $U_x = 25km/h$ and $R = 8m$.

During the experimental process, we use linear vehicle dynamic model combining with the method for linear wheel [4] as the base predictive model to compare the evaluation with the proposed model, and at the same time provide steady state information. The predicted horizon length is 0.5s with 30 steps, using the front lateral force in the simulation as the input of the predictive model, which can generate series of vehicle predictive-state.

In the case of scenario 1 simulation, with a constant radius of $R = 30m$ and velocity $U_x = 45km/h$, lateral acceleration can be up to $6m/s^2$, steering angle is smaller than 5^0 in point 1 and point 3, as well as a little bit higher at point 2 (Figure 4a). Therefore, assuming small angles are grounded, both linearization models accurately predict the states of the vehicle. Nevertheless, the predictive slip side β at point 1 of the reference linearization model starts deviating from the nonlinear value.

In case of simulating-based scenario 2, with velocity $U_x = 25km/h$ and maximum lateral acceleration of $9m/s^2$, the steering angle is larger than 5^0 in point 1, point 2 and point 3 (Figure 4b), especially in point 2. Assuming that the small angle expires with such conditions, the predictive errors of the rear wheel force, the side slip and the slip coefficient for the increasing reference model with the number of steps at point 2. Simultaneously, the linearization model with steady state information still accurately predicts the vehicle condition.

Table 1. Information of parameter values used for simulation

Parameter	Index	Values
Vehicle mass	m	1500kg
Front axle length	l_f	1.1m
Rear axle length	l_r	1.6m
First corner stiffness	C_{α_f}	48N/rad
Rear corner stiffness	C_{α_r}	36N/rad
Yaw inertia	I_{zz}	1450kg.m^2

Friction coefficient	μ	0.9 n/a

In order for the performance of the proposed motion controller is validated, we conduct the experiment in the direction of simulation. The simulation process is carried out with a full-fidelity dynamic model. In Table 1 we can see the path and the vehicle's parameters. The path for the simulated driver controller to monitor is 1 the loop is 450m long and is created by the method in the study [8]. At the same time, this desired path is transferred as a set of curves that change the distance in an anti-clockwise direction along the path, longitudinal velocity and desired lateral acceleration profile are produced by the proposed velocity controller in [7]. The roundabout varies from $-0.05m^{-1}$ to $0.05m^{-1}$, longitudinal velocity varies from $13m/s$ to $30m/s$ and the lateral acceleration varies from $-10m/s^2$ to $10m/s^2$.

The simulation process is carried out with 3 motion controllers by different predictive models, including: linear controller, initial controller and proposed controller. In which the steering angle, the linear vehicle and wheel models are used by the linear controller as the control input. The original controller uses the vehicle dynamic model and the nonlinear wheel as shown. Another difference is that the controller used as a reference uses the lateral deviation e and the initial deviation $\Delta\psi$ as the reference state. At the same time, the vehicle stability limit and the longitudinal controller are used for all situations.

Table 2. Simulation results of 3 controllers

	$\lvert\bar{e}\rvert$	$\sigma(\lvert e\rvert)$	$\max(\lvert e\rvert)$
Linear controller	2.6m	3.3m	10.5m
initial controller	0.7m	0.9m	5.2m
proposed controller	0.65m	0.7m	5.2m

The simulation results (Table 2) show that the maximum value of the absolute lateral deviation $\max(|e|)$, the standard deviation $\sigma(|e|)$ and the absolute lateral deviation $|\bar{e}|$ of the linear controller are much larger than the other two controllers. Furthermore, the simulation results demonstrate that the steering angle of the linear controller continues to increase until it gets to the maximum level, as well as the lateral deviation goes on increasing. The cause of this phenomenon is due to the linear model cannot predict the lateral force when the wheel reaches the non-linear region. On the contrary, with the proposed dynamic model, the controllers can keep the lateral deviations within a small range. This proves that the proposed linear method in part 2 can keep nonlinear characteristics of the vehicles even in high velocity conditions.

The performance part of the initial controller and the proposed controller are quite close. However, the value $||\bar{e}|$ and $\sigma(|e|)$ of the proposed controller is lower, but the max value $(|e|)$ is nearly the same. Thus, at a certain time, the front lateral force needed to reduce the minimum deviation that has overcome the friction force can be used and there will be no other problems when the motion controller can do to take the vehicle operation back to the desired motion trajectory with large lateral acceleration.

V. CONCLUSION

Solution for building motion tracking system with model predictive control based on a linear model that allows to track the vehicle motion being described in the article, the proposed motion controller can track the desired direction desired accurately at high velocity with large lateral acceleration. Including steady-state information and prediction in models, the proposed linear method can precisely maintain non-linear characteristics of the wheel and vehicle models. In addition, the wheel model is linearized to accurately describe the vehicle motion at high velocity and through the simulation process done to increase the reliability of the proposed solution. Assumptions and simulation cases with control reference states are based on linear models. The motion controllers with MPC use direct deviation instead of the head direction as the control state to eliminate tracking deviations. Therefore, we develop an improved model predictive controller with direct deviation and certain constraints.

Finally, by the linear controller comparison method, the results of the simulations prove that the proposed MPC controller with linear model can maintain the deviation within small range, even under large lateral acceleration conditions. The analytical results indicate that the motion controller with direct deviation minimizes the average of the absolute lateral deviation in comparison with the controller of the head deviation. This proves that the proposed path-tracking controller can ensure accurate tracking and steady-state of vehicles under high-velocity into the problems for autonomous vehicle.

The main aim of this solution is to support for the building of autonomous vehicles a safe solution. In the future, in order to complement it effectively, the settings simulated experimentally will be transferred into the real environment with experimental vehicles fully equipped with sensors, and when experimented in reality, some factors will be complemented to analyze the stability of the system so that the behavior of traffic participation is predicted more accurately.

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