ICMI STUDY 24

CONFERENCE PROCEEDINGS

School Mathematics Curriculum Reforms: Challenges, Changes and Opportunities

November 25-30, 2018

Tsukuba, Japan

University of Tsukuba

Editors: Yoshinori Shimizu and Renuka Vithal

International Commission on Mathematical Instruction

筑波大学
University of Tsukuba

The Twenty-fourth ICMI Study

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ICMI Study 24
SCHOOL MATHEMATICS CURRICULUM REFORMS: CHALLENGES, CHANGES AND OPPORTUNITIES
Tsukuba, 26-30 November 2018

PREPARING PRESERVICE TEACHERS TO TEACH MATHEMATICAL LITERACY: A REFORM IN A TEACHER EDUCATION PROGRAM

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To meet the demand of incoming school curriculum reform focusing on competency-based learning in Vietnam, this paper reports on an innovation project on developing secondary mathematics preservice teachers' (PSTs) mathematical literacy and preparing them to teach mathematics contextually. We developed curriculum and studied the effectiveness of the implementation to a secondary mathematics PST education program that integrates mathematical literacy (ML) in methods courses. The courses offer the PSTs opportunities to experience ML as active learners and prepare them to teach ML. The preliminary results on a project-based modeling task show that the PSTs begin to develop an understanding of ML when they take real-life considerations into account in solving the authentic problem. Discussion about the tension between simplifying models and reflecting the real problem, and directions for future study are suggested.

INTRODUCTION

This paper reports on an innovation project on developing secondary mathematics preservice teachers' (PSTs) mathematical literacy (ML) to prepare them to teach mathematics contextually. This initiative is to address the incoming school reformed curriculum, which will be implemented in 2019. The curriculum focuses on competency-based learning (Vietnam Ministry of Education, 2018) and emphasizes the close relationship between mathematics and the real world. Teachers serve as an agent to make changes in their classrooms, and we argue that PSTs should be equipped with the skills to teach mathematics contextually. Therefore, our ongoing project strives to (a) investigate the process to prepare PSTs to teach mathematics contextually that meets the demand of the reformed school curriculum and (b) document the influences, successes, or failures of the implementation on PSTs' knowledge and practice. In this paper, we will address how the PSTs' mathematical literacy understanding is evident through their work on a project-based task in this innovative program.

SCHOOL REFORM – COMPETENCE-BASED CURRICULUM

The current national school mathematics curriculum in Vietnam, started in 2002, rarely highlights the relationship between mathematics and the real world and does not mention mathematical modeling or mathematical literacy explicitly. To meet the demands of the changing society, a reform in curriculum and textbooks following a competency-based learning will begin to be implemented in 2019 (Vietnam Department of Education, 2018). In this reformed curriculum, mathematical modeling is featured as one of the five competencies that involve communication, mathematizing, reasoning and argument, solving problems, and using mathematical tools. The curriculum indicates that it is necessary for students to use mathematics in everyday life. The curriculum underscores real-world contexts in each lesson, and students are required to apply mathematical knowledge they learned to solve real problems as well as to understand the meaning of mathematical knowledge in the real world. The reformed curriculum will provide opportunities for students to experience and apply mathematics to real-life situations and build the connection between mathematics and reality. However, it is challenges for teachers (including PSTs) to teach mathematics in the way the curriculum intended, as they rarely have such experience as a learner and are not trained to teach mathematics contextually. Therefore, we revamp the mathematics teacher education curriculum, implementing the innovation, and research its effectiveness.

CONCEPTUAL FRAMEWORKS FOR REFORMING A PST EDUCATION PROGRAM

Mathematical literacy

ML is an individual's ability to understand and use mathematics in a variety of contexts, including everyday life, professional, and scientific settings. Mathematics serves as a tool to describe, explain, and predict phenomena (The Organization for Economic Cooperation and Development [OECD], 2013). In turn, individuals appreciate the role mathematics plays in the world and how it prepares them to be constructive citizens and make well-founded judgments and decision. Additionally, OECD (2013) utilizes the mathematical modeling cycle (cf. Kaiser & Stender, 2013) to describe students' actions when facing challenges that require them to apply mathematics, including individuals' capacity to formulate, employ, and interpret mathematics in a variety of contexts.

Several researchers and mathematics educators highlight the difference between mathematics and ML and argue that some people who are good at mathematics are not necessarily good at ML (e.g., Steen, 2001). The focus on developing ML might be different from developing mathematical understanding. For example, whereas the aim of developing school mathematical understanding is to help students climb the ladder of abstract structure, ML is anchored in data that are derived from the empirical world. In addition, school mathematics tends to develop school-based knowledge, but ML involves mathematics acting in the world (Steen, 2001). It is important to note that ML used in this context is not limited to understanding and applying arithmetic but the abilities to use different mathematical knowledge, which might include advanced mathematics. Moreover, ML includes not only the skills and knowledge but also the beliefs, dispositions, and habits of mind people need to engage effectively in quantitative situations in life and work (International Life Skills Survey, 2000). The ML concept serves as a foundation to help PSTs make the connection between mathematics and real life.

Knowledge for teaching mathematics

Teacher knowledge is an important predictor of student achievement because a mathematics teacher's decision-making in class is a function, in part, of her/his knowledge (Schoenfeld, 2010). Educational researchers have conceptualized knowledge for teaching to include subject matter knowledge and pedagogical content knowledge (Shulman, 1987). In particular, pedagogical content knowledge (PCK) refers to:

> The most powerful analogies, illustrations, examples, explanations, and demonstrations—[. . .] the most useful ways of representing and formulating the subject that make it comprehensible to others.... Pedagogical content knowledge also includes an understanding of what makes the learning of specific topics easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them. . . (Shulman, 1987, p. 7)

This conceptualization of teacher mathematical knowledge informs us to provide the PSTs with opportunities to learn (OTL) ML and to teach ML to their future students. Therefore, we create opportunities for the PSTs to experience ML as active learners and engage in developing their knowledge/skills to teach ML through teaching tasks such as analyzing curriculum, selecting, adapting tasks, and using appropriate approaches to teach.

RESEARCH DESIGN

Research and Data Collection

This ongoing two-year project ran with a cohort of 120 PSTs. The cohort started participating in the project in September 2017, at the beginning of their third year in the program. We will continue following them to their last school placement. We adopted a design-based research (Cobb et al., 2003) that involves continuing data collection and data analysis, and curriculum development and implementation. First, we identified gaps related to ML in the current mathematics methods courses in the program. The current program indicates limited opportunities for PSTs to experience ML as learners and to develop PCK for teaching ML. In mathematics methods courses, the opportunity to learn in ML is limited (1.2 % of total training time) to an introduction to mathematical modeling and the Program for International Student Assessment (PISA). In 2017, we collected empirical data on the PSTs' opportunities to learn ML and their beliefs about mathematics and mathematics teaching and learning. The initial data analysis sheds light on our curriculum development, focusing on PSTs' mathematics methods courses and PSTs' field experience (school placement), and how to change PSTs' disposition/attitudes towards the subject.

In addition to measuring OTL and beliefs, we captured PSTs' modeling competencies as a proxy of their ML by using both a multiple-choice test and open-ended word problems. We adopted a research-based multiple-choice test (Haines et al., 2002) to measure the PSTs' understanding of ML when they started the methods courses in the program. This tool was developed measuring aspects of modeling competencies, which was administered to the PSTs individually within 60 minutes. We also got the PSTs to work in pairs for 120 minutes on open-ended tasks focusing on four content areas: shapes, quantity, data and chance, and change. The data provided us with information about the PSTs' current content knowledge related to ML and the weakness and strength the PSTs have prior to the methods courses. All the data were used to incorporate the opportunities to learn ML.

We have conducted interviews (task-based and stimulated recall). Other data sources include notes from classroom observations, PSTs' reflection on their placement related to ML, and their lesson plans. In 2019, the project will finish with a post measure of PSTs' ML OTL, beliefs, and modeling competencies. Post interviews will be conducted on some participants (cases). Table 1 summarizes the timeline of data collection.

Curriculum Development and Implementation

Opportunities to learn and teach ML were incorporated into four methods courses: Mathematics Teaching Methods and Assessment of Mathematics Learning (Semester 1 of 2017-18), Mathematics Curriculum Development and New Trends in Mathematics Teaching and Learning (Semester 2 of 2017-18). Additionally, PSTs were asked to reflect on ML when they were at their school placements. The first placement was mainly focusing on observing real classrooms and planning mathematics lessons but not implementing the lessons. In this instance, the PSTs were asked to reflect on how the observed lessons offer ML OTL and nominate their one best lesson plan that incorporates ML. In the second placement when PSTs will plan and implement their lessons in real classes, they will be asked to report on how they incorporate ML into the classes as well as reflect on the challenges/success they have when teaching ML.

Content	**Pre**	**Curriculum Implementation and Data collection**	**Post**
	• OTL ML measures (Individual) • Beliefs about mathematics and mathematics teaching and learning (Individual) • Multiple choice modeling test – Open-ended modeling tasks (Pairs) • Stimulated recall interviews about their OTL and beliefs • Stimulated recall interviews on modeling competencies	• Curriculum Implementation: Mathematics methods courses School Placement • Artefacts collections – Student works and presentations on ML tasks • Classroom observations	• OTL ML measures (Individual) • Beliefs about mathematics and mathematics teaching and learning (Individual) • Multiple choice modeling test – Open-ended modeling tasks (Pairs) • Special-cased interviews on their experience of the program
Timeline	09/2017	09/2017-05/2019	05/2019

Table 1: Project timeline and data collection

First, we exposed PSTs to tasks that offer rich opportunities to engage in ML as active learners. ML tasks have been integrated into mathematics methods courses, which range from standard applications to true (authentic) modeling problems (Tran & Dougherty, 2014). These tasks were adapted from research (e.g., PISA) to fit in the context of Vietnam. Some tasks were created based on the project team's experience with the training program and understanding of the local context, such as designing birthday cake boxes and designing the Hue University of Education parking lots for staff and students. We scaffold PSTs' experience with ML tasks by introducing them with increasing levels of authenticity tasks (Palm, 2009; Tran et al., 2016) that were solved within different time periods, such as several tasks in one session (Semester 1 of 2017-18 academic year), one task in a session (Semester 1 of 2017-18), and project-based tasks that last for several weeks (Semester 2 of 2017-18 school year). These tasks necessitate the use of realistic considerations, not merely mathematical tools. We aimed to help the PSTs to experience revising model and validating process as they went through the modeling cycle.

Second, we aimed to prepare PSTs with PCK to teach ML. In their third year of the program, PSTs were introduced to the modeling cycle (OECD, 2013) to inform phases that students generally go through when solving modeling problems and to reflect on the process of solving ML tasks. In Semester 2 of 2017-18, the PSTs were exposed to knowledge about ML and how to incorporate ML into the current curriculum. PSTs analyzed current school curricula to investigate how ML was introduced in the documents and contrasted with the reformed curriculum. They also explored how ML was emphasized in curricula from other countries. PSTs were asked to plan a lesson that integrates ML into the content specified in the curriculum. In Semester 1 of their fourth year, PSTs will be asked to analyze tasks based on the modeling cycle and the level of authenticity and then adapt them to incorporate into real lessons. In addition, they will analyze student works on modelling tasks and how to evaluate them as an assessment practice.

Project-based Task

In Semester 2 of 2017-18, PSTs were asked to work on the following task: "Currently, in our university campus, there are five parking regions that are close together which look quite messy. Can you design a parking lot for the university to solve the current issue so that it looks neat?" This task was similar to tasks found in literature, yet the uniqueness is that vehicles parked in this task include cars, bikes, electric bikes, and motorbikes, not just cars or motorbikes. PSTs were asked to work on this project for four weeks in groups of 4-5 and report to the class at Week 4. Students presented weekly on their progress of the project to get feedback/questions from peers (not in their groups) and the lecturers to improve their reports. They submitted their written report and gave a presentation to the class. The data for this task included their written report and their presentations.

Data Analysis

To evaluate the initial success of the implementation, we focused on preliminary results regarding different ways the PSTs approached an authentic modeling task of designing a parking lot for the university. A total of 15 written reports were collected. Especially, we looked for (a) evidence PSTs took realistic concerns into account (data, information, technical considerations) when designing the university parking lot and (b) their experience in different phases of modeling cycle when working on the task. We identified how the PSTs transferred from real life to mathematical problems and what variables they took into account to formulate mathematical models. We examined how they solved the problems and interpreted them back to the real-life issues.

PRELIMINARY RESULTS

A preliminary analysis shows that the PSTs formulated two mathematical problems or a combination of them: (a) design parking lots based on the information about the number of vehicles, and (b) find the cost to build the parking lots. The analysis reveals that the PSTs used a combination of arithmetic and proportions as main tools on this task. Some used sampling and data collection techniques to estimate the number of vehicles and used direct measurement and area formulas. Some built regression models to predict the cost.

In this paper, we report two samples from PSTs. The samples were chosen to (a) highlight the PSTs' considerations of real-life issues and the collecting of empirical data (measurement of the parking lot, surveying numbers of each of the vehicles) and (b) represent different mathematical tools the PSTs used to solve relevant mathematics problems formulated from real-world problems (e.g., arithmetic, advanced mathematical tools such as linear programming).

Surveying The Number of Vehicles and Designing Parking Lots

Group 1 specified real-life problems to address the issues messiness of the parking in Hue University of Education, when more people use vehicles to come to university as a result of advancing standards of life. They evaluated the quality of Hue University of Education's current parking and provided a plan for building the new facility with given funding. When attacking this problem, they found information about the number of vehicles present daily at the university and areas available for parking, which were the two sub-mathematical/statistical problems formulated from the real issue.

They searched the university website (http://www.dhsphue.edu.vn) for information about the number of staff members and students. However, the data might not reflect exactly the number of vehicles, which was the main variable to consider when solving the problem (validating). The group then surveyed the number of vehicles of each type on four random weekdays. When collecting the data from four days, on average, they found the percentages of vehicles in each of the parking lots: H (50%), DEG (40%) and GV (10%). In addition, they estimated the number of vehicles for each type as: motorbikes (60%), electric-bikes (15-17%), regular bikes (20-22%), and cars (2-3%). They estimated 1500 vehicles in the student parking lot and 180 in the staff parking lot. Therefore, they decided to design two parking spaces, one for students and one for staff. The student parking lot could hold 900 motorbikes, 250 electric-bicycles, and 350 bicycles. The staff parking lot could hold 135 motorbikes and 45 cars. During this process, the students used sampling and surveying as mathematical/statistical tools to collect data. Moreover, they utilized knowledge about proportion and percentages to decide the capacity for each type of vehicle in each parking lot. On the second sub-problem, the PSTs measured the sizes of current parking lots by applying their knowledge about the area. They drew a floor plan with specified dimensions for the parking lots.

The PSTs then searched for dimensions of each vehicle type to decide the appropriate space for them using a rectangular model and compared the area of the models to those of the real parking lots proportionally. They found that one-story parking lots would not be sufficient to meet the demand of the space for all vehicles; therefore, they needed to look for an alternative design. The PSTs investigated parking lots in other universities and those of a supermarket in the city to look for parking designs and how to operate the parking. As they found that no students travel to the university by cars, they decided to create one parking lot for staff and two for students. After collecting all relevant data, they designed a two-story parking that reserved one story for 45 cars and the other for motorbikes. Particularly, with their survey of car dimensions and spaces between two cars (length: 5,5 m, width: 2,3-4 m and the gap: 4-6 m), they figured out that the area of the parking should be about 36*30 (m^2).

For the students' parking lots, this group revamped the model of current parking lots by including specified dimensions for each row, taking into account the dimension of bikes, motorbikes, and electric bikes with the length of 2m and width of 0.8m. The distance between two consecutive rows is 1.8m. Therefore, they used a 2-meter square for each vehicle in these parking lots. They worked out the number of vehicles for each of the parking lots in the university and checked if the lots meet the demands of student vehicles from their survey (Figure 1). After finishing these sub-problems, they determined the cost to build such parking lots. They then submitted their findings and presented their plans to the class.

Focusing on predicting the cost of building parking lots on the number of vehicles

Group 2 surveyed the number of vehicles on three random days and found 1600 vehicles per day (motorbikes, electric-bikes, regular bikes) for both staff and student. This group measured the sizes of parking lots and calculated areas. They also decided that one of the parking lots needed to be two-story. They decided to build three parking lots: one two-story and two one-story that would connect to the three entrances into the University: G32, G34, G36. Additionally, they formulated a mathematical problem to predict the cost of the parking when knowing the number of vehicles.

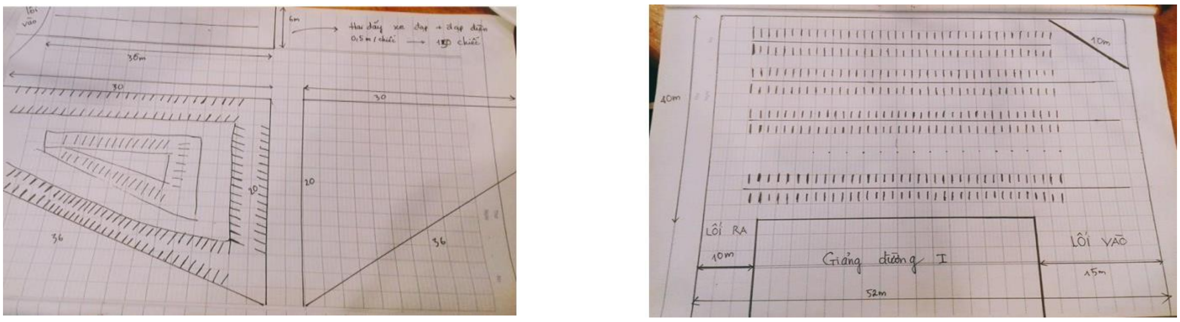

Figure 1: Layout of vehicles into roles in two parking lots

Based on the information about the cost of materials and relevant equipment needed to operate the parking lots (e.g., camera) and the cost to demolish the current parking lots in the University, they recorded the data on a table. The data were based on the following variables: the money to demolish the current parking lot, how much of the old infrastructure could be reused, the area of the parking lots, the number of stories, and the number of vehicles in each of the parking lots. They then graphed the data in a coordinate plane including one axis for the number of vehicles and the other the cost (in Vietnamese dong). They created a power function as an approximation for the collected data to come up with a model. The coefficients were an estimation based on the data, without checking if the models were good for prediction, or a regression model to minimize the total sum of square deviations (Figure 2). The two models follow:

Two-story parking lot at G34 for staff:

C = $\boldsymbol{C_{32}} + \boldsymbol{C_{34}} + \boldsymbol{C_{36}} = 0{,}\mathbf{918}.\boldsymbol{x_{32}}^{\mathbf{0{,}95}} + 2{,}05.\ \boldsymbol{x_{34}}^{\mathbf{0{,}95}} + 1{,}22.\ \boldsymbol{x_{36}}^{\mathbf{0{,}83}}$ (*x*.. is the number of vehicles in the parking .., and C.. the cost to build parking lot ..)

Two-story parking lot at G32 for students and keep the staff G34 parking lot as is:

Model 2: C = $\boldsymbol{C_{32}} + \boldsymbol{C_{36}} = 1{,}28.\ \boldsymbol{x_{32}}^{\mathbf{0{,}98}} + 1{,}22.\ \boldsymbol{x_{36}}^{\mathbf{0{,}83}}$

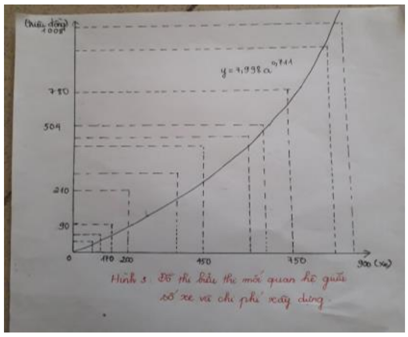

Figure 2: Power functions to predict the cost for one parking

DISCUSSION

This ongoing project is in the process of implementing the innovative curriculum focusing on the developing PCK for the PSTs to teach ML. We have not collected the post data to investigate the effectiveness of the program. However, at this stage, the data show that the PSTs started to experience mathematics in a different way--not merely considering real-world contexts as a cover, which is easily stripped out to reveal the mathematics. Additionally, the PSTs experienced the uncertainty when using mathematics to solve problems they encounter in their lives. However, opportunities to discuss the difference between their estimations of vehicles were not taken, which could be powerful for validation. When predicting the cost to build parking lots, the PSTs need to balance how much they simplify the model so that they can formulate a problem that is solvable versus how to develop a model that is sophisticated enough to capture the real world, yet challenging to solve with their current mathematical knowledge. The PSTs were not familiar with regression models in prediction and unsure how to evaluate the goodness of their model; such findings calls for possibly having collaborations between mathematics educators and mathematicians who are responsible for training the students. A question emerged is what the program would look like if the mathematicians take an ML perspective when teaching their courses: how could PSTs' mathematical knowledge be strengthened?

Within the scope of this project, we investigated how PSTs' knowledge changed during the implementation of the innovation. However, it is challenging to discuss the impact of the implementation on their teaching practices because the PSTs have not had opportunities to teach in a real classroom. Future studies could build on this project and follow the PSTs in their second placement (teaching) and their first years of teaching to see how the training changes their practices.

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