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SURFACE PLASMON-POLARITONS AND TRANSVERSE SPIN ANGULAR MOMENTUM AT THE BOUNDARY OF A HYPERBOLIC METAMATERIAL WITH AN ARBITRARILY ORIENTED OPTIC AXIS

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The features of plasmon-polaritons excited at the boundary of a hyperbolic metamaterial and a dielectric are studied in the case where the optic axis is oriented at an angle to the interface. It is shown that in the metamaterial the phase velocity of the plasmon-polariton is directed at a certain angle to the boundary. The possibility is established and the conditions are determined for localization of plasmon-polaritons at the boundary of both type I and type II hyperbolic metamaterials. A surface plasmon-polariton localized at the interface of an isotropic dielectric and a hyperbolic metamaterial is shown to have transverse spin of a magnitude that depends on the wavelength of the exciting radiation, the orientation of the optic axis, and the dielectric properties of the adjoining media.

Keywords: *localized plasmon-polariton, spin momentum, hyperbolic metamaterial.*

Introduction. A rotatory light beam (e.g., Laguerre–Gauss, Bessel–Gauss) propagating in a specified direction with an electric field amplitude $E(\rho, \varphi) = E_0(\rho) \exp(im\varphi)$, where ρ and φ are spherical coordinates and m is the azimuthal phase index (topological charge), has a spin angular momentum (SAM) associated with its polarization, as well as an orbital angular momentum (OAM) that depends on the profile of the wave front [1, 2]. Each photon in a rotatory beam of this kind carries an orbital angular momentum equal to $m\hbar$. When rotatory beams interact with microscopic objects, SAM and OAM are transferred to particles and can be used to control them.

Evanescent waves, which develop under conditions of total internal reflection in an optically less dense medium, differ fundamentally from propagating waves. Their spin angular momentum is nonzero and perpendicular to the wave vector [3, 4]. Transverse SAM originates in the rotation of the electric field vector of an evanescent wave in the plane of propagation. The features of transverse SAM have been studied for evanescent waves at the boundary of isotropic media [3, 4].

It has been shown [5] that transverse spin momentum can exist in plasmon-polariton fields excited at the boundary of a type II hyperbolic metamaterial (HM) with its optic axis perpendicular to its boundary. In this paper, we study the effect of changes in the orientation of the optic axis of a hyperbolic metamaterial on the conditions for existence of plasmon-polaritons and the transverse spin associated with them at its boundary. As an example we examine an HM based on multilayer metal-dielectric structures.

Surface Plasmon-Polaritons at the Boundary of an Isotropic Dielectric-Hyperbolic (Uniaxial) Metamaterial with an Arbitrarily Oriented Optic Axis. We consider p-polarized surface waves propagating along the x axis in a plane separating two semi-infinite media: an isotropic dielectric with dielectric constant ε and an HM with its optic axis oriented at an angle θ to the z axis, which is perpendicular to the interface. It can be shown that the HM is characterized by a dielectric tensor $\hat{\varepsilon}$ which can be written for the coordinate system used here as follows:

$$\hat{\varepsilon} = \varepsilon_{xx}\mathbf{e}_x \otimes \mathbf{e}_x + \varepsilon_{\perp}\mathbf{e}_y \otimes \mathbf{e}_y + \varepsilon_{zz}\mathbf{e}_z \otimes \mathbf{e}_z + \varepsilon_{xz}(\mathbf{e}_x \otimes \mathbf{e}_z + \mathbf{e}_z \otimes \mathbf{e}_x)\,, \quad (1)$$

where $\varepsilon_{xx} = \varepsilon_{\perp}\cos^2\theta + \varepsilon_{\parallel}\sin^2\theta$, $\varepsilon_{zz} = \varepsilon_{\parallel}\cos^2\theta + \varepsilon_{\perp}\sin^2\theta$, $\varepsilon_{xz} = (\varepsilon_{\parallel} - \varepsilon_{\perp})\sin\theta\cos\theta$; $\varepsilon_{\parallel}$ and $\varepsilon_{\perp}$ are the longitudinal (along the z axis) and transverse (along the x, y axes in the plane of the interface) principal dielectric constants; $\mathbf{e}_x$, $\mathbf{e}_y$, and $\mathbf{e}_z$ are the unit vectors for the coordinate system; and ⊗ denotes the dyadic product.

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The field vectors in the dielectric (d) and metamaterial (m) are written in the form

$$\mathbf{F} = \begin{cases} \mathbf{F}^d e^{\kappa_d z + iqx - i\omega t}, & z < 0, \\ \mathbf{F}^m e^{-\kappa_m z + iqx - i\omega t}, & z > 0, \end{cases} \quad (2)$$

where $\mathbf{F} = \begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix}$; $\mathbf{E}$ and $\mathbf{H}$ are the electric and magnetic field vectors (for p-polarized waves $\mathbf{E} = (E_x, 0, E_z)$ and $\mathbf{H} = (0, H_y, 0)$); $\kappa_{d,m} > 0$ and q are the damping constant and longitudinal wave number of the surface plasmon-polariton with $\kappa_d^2 = q^2 - k_0^2\,\varepsilon_1$ and $k_0 = \omega/c$; ω is the cyclical frequency of the electromagnetic wave; and c is the speed of light in vacuum. For Eq. (2) the Maxwell equations yield the following for the electric and magnetic field vectors in the dielectric and the HM:

$$\mathbf{E}^d = -\frac{A_0}{\varepsilon_1}\left(i\,\frac{\kappa_d}{k_0}\,\mathbf{e}_x + \frac{q}{k_0}\,\mathbf{e}_z\right)e^{iqx+\kappa_d z}, \qquad \mathbf{H}^d = A_0\mathbf{e}_y e^{iqx+\kappa_d z}. \quad (3)$$

$$\mathbf{E}^m = (A_0/sk_0)[i\kappa_m\varepsilon_{zz} + q\varepsilon_{xz})\mathbf{e}_x - (q\varepsilon_{xx} + i\kappa_m\varepsilon_{xz})\mathbf{e}_z]\exp(iqx - \kappa_m z), \quad \mathbf{H}^m = A_0\mathbf{e}_y\exp(iqx - \kappa_m z). \quad (4)$$

Here $s = \varepsilon_{xx}\varepsilon_{zz} - \varepsilon_{xz}^2$; A_0 is the amplitude; and the phase factor $\exp(-i\omega t)$ has been omitted. On substituting Eq. (4) in the Maxwell equation rot $\mathbf{E} = -(1/c)\partial\mathbf{H}/\partial t$, we find

$$\kappa_m^2 = aq^2 - bk_0^2 + 2iqd\kappa_m, \quad (5)$$

where $a = \varepsilon_{xx}/\varepsilon_{zz}$, $b = s/\varepsilon_{zz}$, and $d = \varepsilon_{xz}/\varepsilon_{zz}$. For angles θ that are small or close to 90^o, ε_{xz} (and, therefore, d) is small, so that according to Eq. (5) the damping constant κ_m is a complex quantity given by

$$\kappa_m = \kappa_{m0} + i\kappa_{m1} = (aq^2 - bk_0^2)^{1/2} + i(\varepsilon_{xz}/\varepsilon_{zz})q. \quad (6)$$

It follows from Eqs. (4) and (6) that the propagation constant for a plasmon-polariton in an HM has a component normal to the interface boundary. This means that in the HM the phase velocity is directed at an angle γ to the boundary of the HM:

$$\tan\gamma = -\varepsilon_{xz}/\varepsilon_{zz}. \quad (7)$$

Equation (7) implies that for both type I and type II HM, it is possible, by choosing the orientation of the optic axis, to reach a condition such that the phase velocity of the plasmon-polariton is directed (1) inward from the boundary of the HM or (2) toward the boundary. Case 1 occurs for a type I HM ($\varepsilon_\perp > 0$, $\varepsilon_\| < 0$) for $\theta > 0$ (a positive angle corresponds to an optic axis positioned between the z and x axes of the chosen coordinate system) and for a type II HM ($\varepsilon_\perp < 0$, $\varepsilon_\| > 0$) for $\theta < 0$. Case 2 occurs for type I (II) HM for $\theta < 0$ ($\theta > 0$). For $\theta \to 0$ (optic axis (quasi)orthogonal to the boundary of the HM) or $\theta \to \pi/2$ (optic axis at a small angle to the boundary surface of the HM), the transverse component of the wave vector of the plasmon-polariton vanishes.

The boundary conditions for the vectors (3) and (4), together with Eq. (6), can be used to obtain an equation for the longitudinal wave number of the plasmon-polariton. In the simplest case, for small d, we have

$$q = k_0\sqrt{\frac{\varepsilon_1\varepsilon_{zz}(\varepsilon_{xx} - \varepsilon_1)}{\varepsilon_{xx}\varepsilon_{zz} - \varepsilon_1^2}} = k_0\sqrt{\varepsilon_{\text{eff}}}. \quad (8)$$

We now analyze the possible existence of a fully localized surface wave at the boundary of an isotropic medium and an HM. Since the amplitude of this kind of field falls off exponentially with distance from the interface, the wave number q should be greater than the moduli of the wave vectors of the waves propagating in the dielectric and the HM, i.e.,

$$\varepsilon_{\text{eff}} > 0,\ \varepsilon_{\text{eff}} > \varepsilon_1,\ \kappa_{m0} > 0. \quad (9)$$

Equations (8) and (9) yield the conditions for localization of a plasmon-polariton on the HM surface:

$$\varepsilon_{zz} > \varepsilon_1,\ \varepsilon_{xx} < 0. \quad (10)$$

For a type I HM, Eq. (10) gives

$$\sin^2\theta > Y_{b1} = \sin^2\theta_{b1} = \varepsilon_\perp/(|\varepsilon_\||+\varepsilon_\perp), \quad \sin^2\theta > Y_{b2} = \sin^2\theta_{b2} = (|\varepsilon_\||+\varepsilon_1)/(|\varepsilon_\||+\varepsilon_\perp). \quad (11)$$

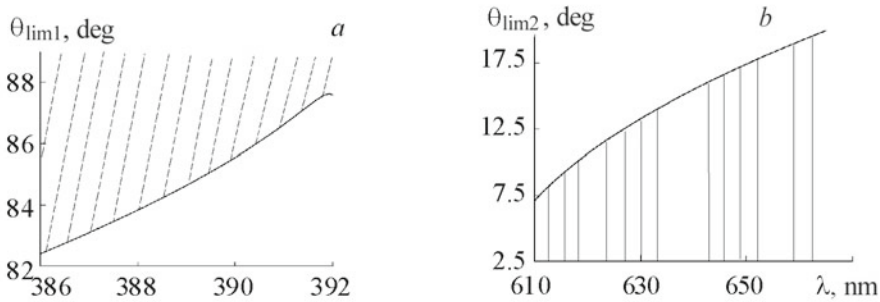

Fig. 1. Spectral dependences of the limiting angles $\theta_{\lim 1}$ (a) and $\theta_{\lim 2}$ (b) that determine the condition for localization of a plasmon-polariton excited at the boundary of a Ti_3O_5/Ag and water metal–dielectric layered medium; the thickness of the metal layers is 20 nm; the filling factor is $f = 0.3$; and the shaded region corresponds to satisfaction of the condition for existence of localized plasmon-polaritons.

Equation (11) yields the condition for existence of localized plasmon-polaritons at the boundary between a type I HM and an isotropic dielectric:

$$|\,\theta\,| > \theta_{\lim 1} = \max\left\{\left|\arcsin\sqrt{Y_{b1}}\right|, \left|\arcsin\sqrt{Y_{b2}}\right|\right\}. \tag{12}$$

For a type II HM, Eq. (10) implies that the excited plasmon-polariton is localized when the following conditions are met:

$$\sin^2\theta < Y_{b3} = |\varepsilon_\perp|/(\varepsilon_\parallel + |\varepsilon_\perp|), \quad \sin^2\theta < Y_{b4} = (\varepsilon_\parallel - \varepsilon_1)/(\varepsilon_\parallel + |\varepsilon_\perp|). \tag{13}$$

Thus, as Eq. (13) shows, when the condition

$$|\,\theta\,| < \theta_{\lim 2} = \left|\min\left\{\left|\arcsin\sqrt{Y_{b3}}\right|, \left|\arcsin\sqrt{Y_{b4}}\right|\right\}\right| \tag{14}$$

is satisfied at the boundary of a type II HM with an isotropic dielectric, a fully localized surface wave (surface plasmon-polariton) can exist, with a field that is damped exponentially on both sides of the boundary.

As an example, we consider an HM based on a Ti_3O_5/Ag metal–dielectric layered structure. In the approximation of an effective medium, for the principal dielectric constants of this structure we have

$$\varepsilon_\perp = (1-f)\varepsilon_d + f\varepsilon_m, \quad \varepsilon_\parallel = [(1-f)/\varepsilon_d + f/\varepsilon_m]^{-1}. \tag{15}$$

Here ε_d and ε_m are the dielectric constants of dielectric and metal layers and f is the filling factor (volume fraction of metal in a cell). The dielectric constant ε_m of the metal is described by a modified Drude formula

$$\varepsilon_m(\omega) = \varepsilon_\infty - \omega_p^2/(\omega^2 + i\omega\Gamma) = \varepsilon_\infty - \omega_p^2/(\omega^2+\Gamma^2) + i\,\omega_p^2\,\Gamma/[\omega(\omega^2+\Gamma^2)],$$

where $\omega = 2\pi/\lambda$ is the cyclical frequency; ω_p is the plasma frequency; ε_∞ is constant; $\Gamma = V_F/l$ is the damping constant; V_F is the Fermi velocity; and l is the electron mean free path in the bulk metal. For the example of silver, $\varepsilon_\infty = 5$, $\omega_p = 14\cdot10^{15}$ s^{-1}, $\Gamma = 32\cdot10^{12}$ s^{-1}, and $V_F = 1.4\cdot10^6$ ms^{-1} [6]. A calculation shows that a periodic Ti_3O_5/Ag layered structure with $f = 0.3$ has the properties of a type I HM in the spectral range $361 \le \lambda \le 392$ nm. Let the layers be deposited on a substrate (e.g., of fused quartz with a dielectric constant of $\varepsilon_1 = 2.1$) in a way such that the axis of periodicity (the optic axis) of the structure is oriented at an angle θ to its input facet. As Fig. 1a shows, the condition (12) is satisfied for angles close to 90° in this case.

A calculation according to Eq. (15) shows that periodic Ti_3O_5/Ag layered structure with $f = 0.3$ has the properties of a type II HM for $\lambda > 600$ nm. Figure 1b shows that the condition (12) is satisfied for small deviations of the optic axis from the normal to the surface of the metal-dielectric structure.

Transverse Spin Momentum of a Surface Plasmon-Polariton at an Isotropic Dielectric-Hyperbolic (uniaxial) Metamaterial Interface with an Arbitrarily Oriented Optic Axis. As Eqs. (3) and (4) imply, the electric field vector of a plasmon-polariton localized in a dielectric in its plane of propagation (xz) has real transverse and imaginary longitudinal components: $\mathbf{k}^d = q\mathbf{e}_x + i\kappa_d\mathbf{e}_z$. Thus, a phase difference of $\pi/2$ appears between the components E_x^d and E_z^2, which causes the electric field vector to rotate in the propagation plane (xz) and produces a transverse component S_y of the SAM [3,4],

$$\mathbf{S} = (1/16\pi\omega)\,\mathrm{Im}\,[\mu^{-1}(\mathbf{E}^*\mathbf{E}) + \varepsilon^{-1}(\mathbf{H}^*\mathbf{H}). \tag{16}$$

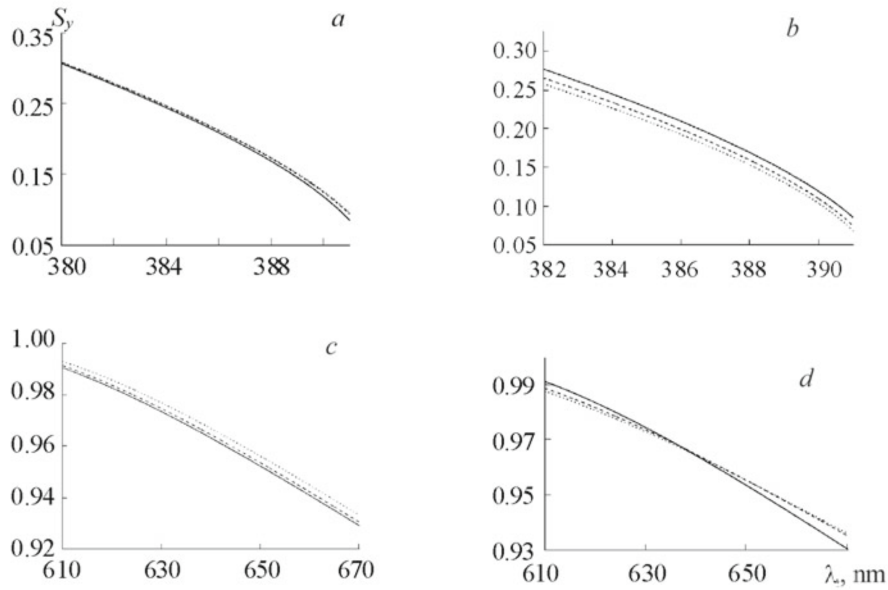

Fig. 2. Spectral dependence of the transverse spin (in units of $\hbar$) for a localized wave formed at the boundary of a Ti_3O_5/Ag metal–dielectric layered medium with a dielectric: (a, c) water; (b, d) water (smooth curve), sunflower seed oil (dashed curve), and BAF 10 optical glass (dotted curve). The thickness of the metal layers is 20 nm, the filling factor is $f = 0.3$, and the optic axis is oriented at an angle of (a) $\theta = 87.8^o$ (smooth curve), 89^o (dashed curve), 90^o (dotted curve); (b) 87.8^o; (c) $\theta = 1^o$ (smooth curve), 3^o (dashed curve), 5^o (dotted curve); (d) 3^o.

Given Eq. (3), from Eq. (16) we obtain an expression for the transverse component of the SAM in the dielectric:

$$S_y = \frac{|A_0|^2}{8\pi\omega\varepsilon_1^2} \frac{\kappa_d q}{k_0^2} e^{2\kappa_d z}, \quad z < 0. \tag{17}$$

Given that $|A_0|^2 = 8\pi\varepsilon_1 k_0^2 W/q^2$, where $W = (\varepsilon_1 \mathbf{E}\mathbf{E}^* + \mathbf{H}\mathbf{H}^*)/16\pi$ is the time averaged energy density of the field, Eq. (17) yields

$$S_y = \frac{W}{\omega\varepsilon_1} \frac{\kappa_d}{q} e^{2\kappa_d z}, \quad z < 0. \tag{18}$$

Equation (18) can be used for calculating the transverse SAM carried by a spatially localized surface wave in a non-absorbing dielectric. Equation (18) implies that S_y depends on properties of the surrounding media. For an absorbing isotropic dielectric medium, in accordance with [7], the expected value of S_y (in units of $\hbar$) is

$$S_y = -2 \frac{\mathrm{Re}\, E_x \ \mathrm{Im}\, E_z - \mathrm{Im}\, E_x \ \mathrm{Re}\, E_z}{|E_x|^2 + |E_z|^2}, \tag{19}$$

where E_x and E_z are the components of the electric field vector $\mathbf{E}^d$. Substituting Eq. (3) in Eq. (19), for $z = 0$ we obtain

$$S_y = 2\kappa_d q/(q^2 + \kappa_d^2). \tag{20}$$

Equation (20) gives the angular momentum acting in the direction of the y axis on a particle located near the HM surface.

Figure 2 shows the spectral dependences of the transverse spin (in units of $\hbar$) for a plasmon-polariton localized at the boundary of a Ti_3O_5/Ag metal–dielectric layered medium with a dielectric (water, oil, glass). It can be seen that S_y varies with wavelength, which indicates that it may be possible to control the transverse spin momentum and, thereby, the force acting on a microscopic particle located in a dielectric near its boundary with an HM by changing the wavelength of light used to excite a surface plasmon-polariton. For the spectral ranges within which this metal–dielectric structure has type I and type II

HM properties, however, the $S_y(\lambda)$ curves are different. Thus, on approaching the wavelengths for which $\varepsilon_\perp > 0$, $\varepsilon_\| < 0$, and $\varepsilon_\| > 0$ from shorter wavelengths, S_y decreases (Figs. 2a and b), but on approaching the wavelengths for which $\varepsilon_\| > 0$, $\varepsilon_\perp < 0$, and $\varepsilon_\perp \to 0$ from longer wavelengths, S_y increases (Figs. 2c and d). In addition, the magnitude of the transverse spin also depends on the orientation of the optic axis and the dielectric constant of the medium adjoining the HM. The shapes of these curves are, however, different in the spectral ranges corresponding to type I and type II HM. In the first case, with increasing θ the steepness of the $S_y(\lambda)$ curve decreases (Fig. 2a), and with increasing ε_1 of the component, S_y decreases within this range of wavelengths (Fig. 2b). In the second case, the transverse spin S_y also increases with increasing θ (Fig. 2c), but the $S_y(\lambda)$ curve becomes less steep with increasing ε_1 (Fig. 2d).

Conclusions. The features of plasmon-polaritons excited at the interface of a hyperbolic metamaterial and a dielectric have been studied for the case in which the optic axis is oriented at an angle to the interface boundary. Formulas are obtained for the complex electric and magnetic field vectors and for the damping coefficients for the fields on both boundaries of the interface. It is shown that the propagation constant of a plasmon-polariton in a metamaterial has a component normal to the interface, so that the phase velocity in the metamaterial is directed at an angle to its boundary. It is found for both type I and type II hyperbolic metamaterials that it is possible to obtain conditions such that the phase velocity of a plasmon-polariton is directed inward from the boundary into the metamaterial or toward the interface by choosing the orientation of the optic axis. The possibility is established and the conditions are determined for localization of plasmon-polaritons at the boundary of both type I and type II hyperbolic metamaterials. A surface plasmon-polariton localized at the interface of an isotropic dielectric and a hyperbolic metamaterial is shown to have transverse spin of a magnitude that depends on the wavelength of the exciting radiation, the orientation of the optic axis, and the dielectric properties of the adjoining media.

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