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Surface Optical Waves at the Boundary of Nonlinear Hyperbolic Metamaterial

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The conditions of excitation of a special surface wave at the boundary of an isotropic medium and a hyperbolic metamaterial with Kerr nonlinearity are found. The main feature of this wave is non-exponential decay of the amplitude when moving off the boundary. The dependencies of the energetic characteristics of the wave on the propagation constant and the depth of penetration inside metamaterial are established and analyzed.

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1. Introduction

It is known that nonlinear surface waves can propagate at the interface between linear and nonlinear media [1, 2]. These waves are of great interest due to their possibility to create effective waveguide inside nonlinear medium owing to the dependence of its dielectric permittivity on the light intensity. The features of surface waves at the boundary of uniform Kerr nonlinear medium are considered in the paper [3].

Over the last decade there have appeared a number of scientific publications devoted to the artificial structures engineered to provide unusual electromagnetic, physical and other properties not found in natural materials [4]. It is reasonable to expect that the large advantages of nonlinear optics may find a new playground in the field of electromagnetic metamaterials, so that a desired nonlinear response can be constructed depending on particular needs and enhanced as compared to conventional nonlinearity in natural materials.

It should be noted that these composite materials can be described by the effective parameters generally different from the parameters of their constituent materials [5].

Among the most promising MMs are hyperbolic metamaterials (HMMs) for which hyperbolic dispersion is realized originating from one of the principal components of the effective electric permittivity tensor of an anisotropic material having opposite sign to two other principal components [6–8]. These HMMs can be practically realized in the form of a stack consisting of subwavelength alternate plasmonic and dielectric layers or a periodic arrangement of metallic nanowires in a dielectric host [9–12]. The greatest advantage of HMMs is that they offer a multi-functional domain to realize novel electromagnetic devices at optical frequencies. Recently, a number of paper related to the nonlinear interactions in optical HMMs appeared. The area of investigations include the nonlinear interaction of meta-atoms through optical coupling, intensity dependent transmission, all-optical modulation, the enhancement of the nonlinear optical response of metamaterials by using nonlocality, and so on [13–20]. In this paper we consider the features of surface wave propagation at the boundary of nonlinear hyperbolic metamaterial. Meanwhile, we assume that HMM is created on the basis of layered periodic structure, the unit cell of which is formed with nanolayers of a dielectrics and metal (gold) [21].

The paper is structured as follows. In Section 2 the generation conditions of the surface wave at the boundary of linear dielectric and nonlinear

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HMM are obtained. The energetic characteristics of this wave are investigated analytically and numerically in Section 3. At the end of the paper the conclusion is given.

2. Condition for existence of surface wave at the boundary of nonlinear hyperbolic metamaterial

We consider s-polarized wave propagating at the interface between two semi-infinite media: dielectric with permittivity ε_1 and hyperbolic metamaterial which is characterized within the effective medium approximation by the permittivity tensor ε

$$\varepsilon = \widetilde{\varepsilon}_t(\overrightarrow{e_x}\cdot\overrightarrow{e_x}+\overrightarrow{e_y}\cdot\overrightarrow{e_y})+\widetilde{\varepsilon}_l\overrightarrow{e_z}\cdot\overrightarrow{e_z}, \qquad (1)$$

where $\widetilde{\varepsilon}_t = \epsilon_t + \varepsilon_{nl}|\overrightarrow{E}|^2$, $\widetilde{\varepsilon}_l = \epsilon_l + \varepsilon_{nl,l}|\overrightarrow{E}|^2$ are the transverse and longitudinal components of permittivity tensor of hyperbolic metamaterial, respectively; ε_{nl} and $\varepsilon_{nl,l}$ are the coefficients of Kerr-type nonlinearity; $\overrightarrow{e_x}$, $\overrightarrow{e_y}$, and $\overrightarrow{e_z}$ are the unit vectors of the coordinate system. Meanwhile, the optical axis z of HMM is orthogonal to the boundary (Fig. 1).

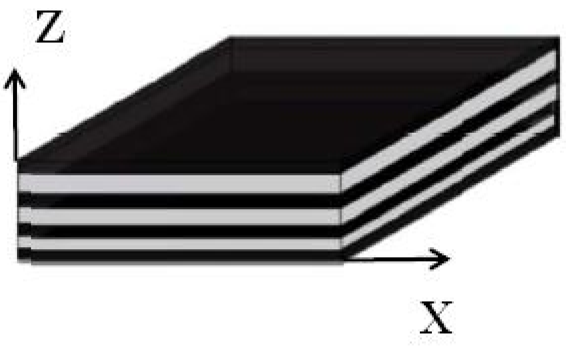

FIG. 1: Metal-dielectric structure.

We shall represent the field inside the dielectric in the form

$$\overrightarrow{E_d} = B_0\overrightarrow{e_y}\exp(\kappa_d z + iqx), \qquad (2)$$

$$\overrightarrow{H_d} = \frac{B_0}{k_0}(i\kappa_d\overrightarrow{e_x}+q\overrightarrow{e_z})\exp(\kappa_d z + iqx). \qquad (3)$$

Here and below the multiplier $exp(-it\omega)$ is omitted, and

$$\kappa_d^2 = q^2 - k_0^2\varepsilon_1. \qquad (4)$$

In Eqs. (2)-(4) $k_0 = \omega/c$, q is the longitudinal wave number, κ_d is the transverse wave number characterizing the localization of surface wave inside the dielectric.

Now we find the field inside the nonlinear hyperbolic metamaterial. For electric field of s-polarized wave we have:

$$\overrightarrow{E_m} = A_m(z)\overrightarrow{e_y}\exp i(qx+\delta), \qquad (5)$$

where the amplitude $A_m(z)$ is dependent on the penetration depth inside the metamaterial , and δ is the phase constant. Let us use the solution of the wave equation for nonlinear uniform medium in the condition of total internal reflection [22] for the amplitude of the field inside hyperbolic metamaterial

$$A_m(z) = A_0\frac{(1+s^2)\exp(-\kappa_m z)}{1+s^2\exp(-2\kappa_m z)}. \qquad (6)$$

Here A_0 is the value of the amplitude $A_m(z)$ at $z = 0$, $s = p \pm (p^2-1)^{1/2}$, $p = a/A_0$, $a^2 = 2\kappa_m^2/(k_0^2\varepsilon_{nl})$, $\kappa_m^2 = q^2 - k_0^2\varepsilon_t$.The magnetic vector inside the hyperbolic metamaterial is represented in the following form:

$$\overrightarrow{H_m} = \frac{1}{k_0}[i(dA_m/dz)\overrightarrow{e_x}+qA_m(z)\overrightarrow{e_z}]\exp(i(qx+\delta)). \qquad (7)$$

Here $dA_m(z)/dz$ is the derivative of the amplitude.

The case $s = p - (p^2-1)^{1/2}$ (related to the nonlinear total internal reflection) has been analyzed in Ref. [22]. In this case the wave decays slowly moving off the input surface of the nonlinear medium.Now we consider the second case when $s = p + (p^2-1)^{1/2}$. Thus, for the penetration depth, for which the amplitude achieves its maximal value, we have:

$$z_m = \frac{1}{\kappa_m}\ln(s). \qquad (8)$$

From the boundary conditions for the fields (Eqs. (2), (3), (5), and (7)) we find:

$$\kappa_d = \sqrt{\kappa_m^2 - A_0^2 k_0^2 \varepsilon_{nl}/2}, \delta = 0. \qquad (9)$$

Thus for excitation of a special surface wave at the boundary of dielectric and nonlinear hyperbolic metamaterial we have the following condition :

$$\varepsilon_1 - \varepsilon_t = A_0^2 \varepsilon_{nl}/2. \qquad (10)$$

In accordance with Eq. (10) the depth where the amplitude of the wave achieves its maximal value is determined by the relation:

$$z_m = \frac{1}{2\kappa_m} \ln \frac{\kappa_m + \kappa_d}{\kappa_m - \kappa_d}. \qquad (11)$$

Using obtained formula, we'll analyze the possibility of excitation and features of special surface wave at the boundary of the nonlinear metamaterial created on the basis of metal-dielectric periodic layered structure. Its elementary cell is formed by linear dielectric (ITO) and nonlinear (with nonlinearity of Kerr-type) metal (Au) layers having permittivities and thicknesses ε_d, d_d, $\widetilde{\varepsilon_m}$, d_m, respectively. It should be noted that permittivity $\widetilde{\varepsilon_m}$ of metallic layer is given by the formula:

$$\widetilde{\varepsilon_m} = \varepsilon_m + \varepsilon_{nl,m} |\overrightarrow{E}|^2. \qquad (12)$$

Here $\varepsilon_{nl,m}$ is the Kerr nonlinear coefficient which equals to $7.71 \cdot 10^{-19}$ m^2/V^2 for gold (Au) [21]. The value of ε_m is determined by Drude model:

$$\varepsilon_m = \varepsilon_\infty - \frac{\lambda^2}{\lambda_p^2 (1 + i\lambda\Gamma/(2\pi c))}, \qquad (13)$$

where ε_∞ is the sum of the interband contributions, λ is the wavelength of optical radiation, λ_p is the plasma wavelength, Γ is the damping constant. For gold nanolayer, for example, we have $\varepsilon_\infty = 9$, $\lambda_p = 136.5$ nm, $\Gamma = 32 \cdot 10^{12}$ s $^{-1}$[4].

Since the Kerr nonlinearity is a weak one, the effective medium approximation still holds for the nonlinear structure. Thus, the main effective permittivities of the metal-dielectric periodic layered structure can be represented as :

$$\widetilde{\varepsilon_l} = [(1-f)/\varepsilon_d + f/\widetilde{\varepsilon_m}]^{-1}, \qquad (14)$$

$$\widetilde{\varepsilon_t} = \widetilde{\varepsilon_m} f + \varepsilon_d (1-f). \qquad (15)$$

Here f is the volume part of the metal layers in the periodic layered structure (filling ratio). Hence, we have

$$\widetilde{\varepsilon_l} = \varepsilon_l + \varepsilon_{nl,l} |\overrightarrow{E}|^2, \varepsilon_{nl,l} \cong (1-f)\varepsilon_{nl,m}/\varepsilon_m \varepsilon_d, \qquad (16)$$

$$\widetilde{\varepsilon_t} = \varepsilon_t + \varepsilon_{nl} |\overrightarrow{E}|^2, \varepsilon_{nl} = f \varepsilon_{nl,m}. \qquad (17)$$

As follows from results of numerical modeling (see Fig.2), for weak light fields when one can neglect the Kerr nonlinearity, for the fillling ratio $f = 0.5$ this structure displays the properties of the I type HMM within the spectral region from 410 nm up to 465 nm.

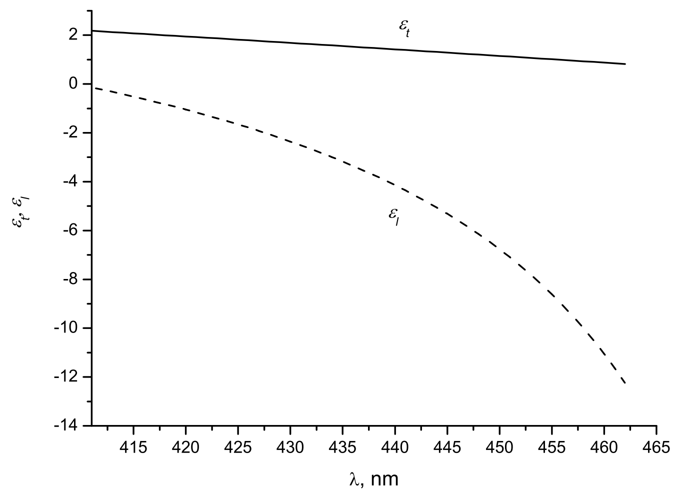

FIG. 2. Spectral dependence of the effective permittivities ϵ_t and ε_l for HMM on the base of ITO/Au layered-periodic structure. Thicknesses of ITO and Au films are 20 nm.

As follows from Eq.(10), the surface wave is excited at the boundary of HMM if $\varepsilon_1 > \varepsilon_t$. In Fig. 3 the spectral dependencies of the value $\Delta\varepsilon = \varepsilon_1 - \varepsilon_t$ for three different bordered media (air, water, and fused quartz) are represented. It is seen

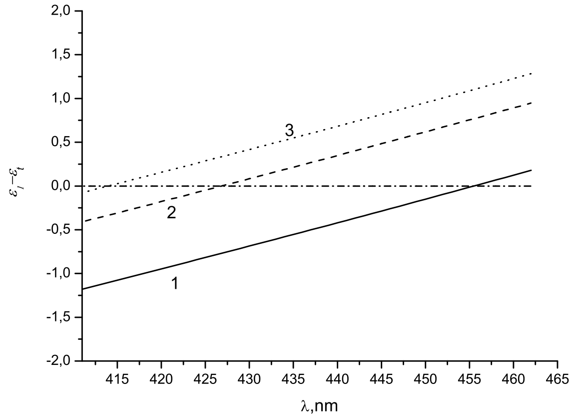

FIG. 3. Spectral dependence of the parameter $\varepsilon_1 - \varepsilon_t$. HMM is formed from ITO/Au layered-periodic structure. Thicknesses of ITO and Au films are 20 nm. Isotropic dielectric borders with the metamaterial is air (curve 1), water (curve 2), fused quartz (curve 3).

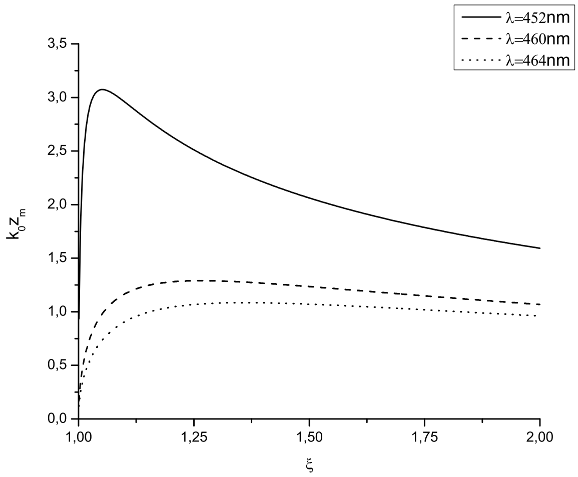

FIG. 4. Dependence of dimensionless parameter $k_0 z_m$ on the propagation constant of nonlinear surface wave $\xi = q/k_0$ excited at the boundary of HMM on the basis of ITO/Au layered-periodic structure and air. Thicknesses of ITO and Au films are 20 nm.

that if the permittivity of dielectric ε_1 increases the spectral region, where the surface wave can exist, becomes larger.

It should be mentioned that, as seen from Fig. 3, a considered surface nonlinear wave can be excited at the boundary of HMM and air. In Fig. 4 the dependence of dimensionless parameter $k_0 z_m$ on the propagation constant of nonlinear surface wave $\xi = q/k_0$ is represented for this case. As is seen from Fig. 4, if the wavelength increases, the maximum of $k_0 z_m(\xi)$ dependence becomes weak.

3. Energetic characteristics of the surface wave

Consider now the problem of the energy transfer by the s-polarized nonlinear surface wave. Using the well-known relations for the Poynting vector $\overrightarrow{P}$ and the density of energy W we can write:

$$\overrightarrow{P} = \frac{c}{8\pi} Re([\overrightarrow{E}\overrightarrow{H}^*]) = \frac{c\xi}{8\pi} A_m^2(z)\overrightarrow{e_x}, \qquad (18)$$

$$W = \frac{1}{16\pi}(\varepsilon|\overrightarrow{E}|^2 + |\overrightarrow{H}^2|) = \frac{\xi^2}{8\pi} A_m^2(z)[1 + \frac{\varepsilon_{nl} A_m^2(z)}{4\xi^2}]. \qquad (19)$$

From Eq.(18) it follows that the Poynting vector has only longitudinal component P_x(parallel to the boundary surface). Meanwhile, this component as well as the density of energy and the ray velocity $\overrightarrow{u}$ of the wave

$$\overrightarrow{u} = \overrightarrow{P}/W \qquad (20)$$

depends on the penetration depth z.

In Figs. 5, 6 there are represented the dependence of the longitudinal component $P_x(z)$ of the nonlinear surface wave and the density of energy $W(z)$ (normalized to their value at $z = 0$) on the distance inside HMM formed on the basis of multilayered nanostructure ITO/Au. We supported that HMM is bordered with air. It is seen that if the distance z from the boundary increases up to z_m the values $P_x(z)/P_x(z = 0)$ and $W_x(z)/W_x(z = 0)$ also increases. For $z > z_m$ these parameters decrease slowly up to zero. It should be noted that for a definite value of z both the longitudinal component of the Poynting vector and the density of energy depend on the propagation constant ξ of the surface wave (see Figs. 5, 6).

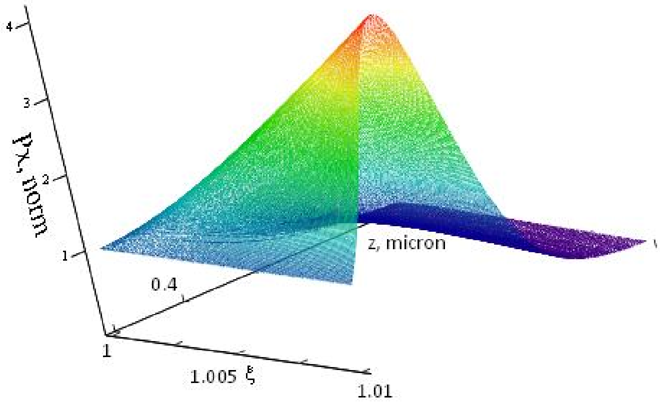

FIG. 5. Dependence of the longitudinal component of the Poynting vector normalized to its value at z=0 on the propagation constant ξ of surface wave and the penetration depth for HMM formed from ITO/Au layered-periodic structure. Thicknesses of ITO and Au films are 20 nm. HMM borders with air. (In colour)

In Fig. 7 the dependencies of the ray velocity on the penetration depth z inside the HMM under consideration and propagation constant ξ are represented. It is seen that at the boundary ($z = 0$) the ray velocity is equal to the phase velocity of the wave $v = c/\xi$. After that with increasing the distance from the boundary up to z_m it decreases and then increases approaching the phase velocity of the wave.

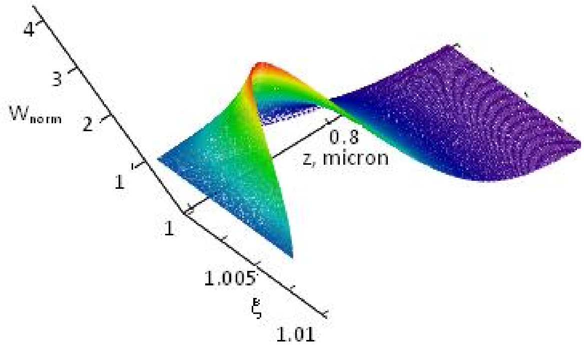

FIG. 6. Dependence of the density of energy normalized to its value at $z = 0$ on the propagation constant of surface wave and the penetration depth for HMM formed from ITO/Au layered-periodic structure. Thicknesses of ITO and Au films are 20 nm. HMM borders with air. (In colour)

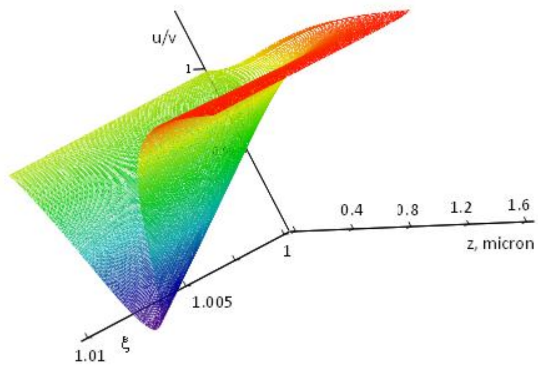

FIG. 7. Dependence of the ray velocity of surface wave normalized to its phase velocity on the propagation constant and the penetration depth for HMM formed from ITO/Au layered-periodic structure. Thicknesses of ITO and Au films are 20 nm. HMM borders with air. (In colour)

4. Conclusions

Features of s-polarized surface wave that can be excited on the boundary of isotropic medium and hyperbolic metamaterial with the Kerr nonlinearity have been studied. The main peculiarity of this wave is non-exponential decay of its amplitude inside the metamaterial when the distance from the boundary increases. The condition for existence of this wave is found and analyzed. It is shown that the value of distance z_m exists for which the amplitude has a maximum. Meanwhile, the parameter z_m depends on the propagation constant of the wave.

Equations for the Poynting vector, the density of energy and the ray velocity of this nonlinear surface wave inside HMM are obtained. Their dependence on the propagation constant and the penetration depth in hyperbolic metamaterial is established. It is shown that when z increases from zero up to z_m the ray velocity inside HMM decreases, after that it increases approaching the phase velocity of the wave.

The obtained results can be used for the development of new methods of diagnostics of metal-dielectric nanostructures based on application of nonlinear surface waves.

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