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Evaluation for half-lives in α-decay chains of $^{309-312}126$ based on semi-empirical approaches

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Abstract

In this paper, we estimated half-lives using semi-empirical formulae for isotopes with $Z = 100 - 126$ in four α-decay chains, which can appear in the syntheses of the $^{309-312}126$ nuclei. The spontaneous fission half-lives were calculated using the Anghel, Karpov, and Xu models, whereas the α-decay ones were predicted using the Viola-Seaborg, Royer, Akrawy, Brown, modified formulae of Royer, Ni, and Qian approaches. We found that there are large differences among the spontaneous fission half-lives estimated using the Xu model and those calculated using the others, which are up to 50 orders of magnitude. The α-decay half-lives also have large uncertainties due to difference in either methods or uncertainties in nuclear mass and spin-parities. Subsequently, there is an argument in determination of α-emitters, especially for the $^{312}126$ isotope. On the other hand, the α-decay half-lives are in the range from a few microseconds ($^{309-312}126$) to thousands of years ($^{257-260}$Fm) in the decay chains. It was found that the half-lives are very sensitive to not only the shell closure but also the angular momentum in the α decay. For experiments, with relatively long half-lives (a few milliseconds), the $^{289-292}$Lv isotopes can be observed as evidences for syntheses of the unknown super-heavy $^{309-312}126$ nuclei. Furthermore, measurements for precise mass, fission barrier, and spin-parity are necessary to improve accuracy of half-life predictions for super-heavy nuclei.

1. Introduction

Half-lives of super-heavy nuclei (SHN) are an important subject of studies for filling gaps in the nuclear chart and understanding nuclear structure. The lifetimes of SHN are mainly determined by the spontaneous fission (SF) and the alpha decay (αD), leading to a need of studies on the competition between these processes. Besides, the alpha decay is one of the most efficient approaches to investigate nuclear properties of SHN. For instance, long αD half-lives reflect shell closures, high fission barriers, and shell effects in SHN [1–6]. Indeed, the SF probability is increased with the decreasing fission barriers and the increasing atomic numbers. The spontaneous fission is, therefore, a significant factor to predict the island of stability of SHN. The SF and αD half-lives are important to determine the limit of stability, which is predicted to be established at $Z^2/A \approx 48$ [7]. Moreover, for syntheses of newly artificial elements, observations of SHN in laboratories strongly depend on their lifetimes and decay mode. Since αD chains are as a reliable tool to discover new SHN [8–15], the accurate determination of decay mode plays a key role in SHN detections. Because beta decay occurring via the weak interaction is slower than SF and αD processes, the longer SF half-life will enable alpha emission from SHN. In such scenario, alpha spectrometers are employed to detect new SHN in measurements. Furthermore, even though the cross sections are large, new elements are difficult to be observed if their lifetimes are less than the measuring timescale (about 1 ms) of recoil separators at recent accelerator facilities [16]. Hence, understanding the half-life and, subsequently, the decay mode is important to evaluate the feasibility of SHN syntheses.

Recently, several attempts for estimating αD and SF half-lives of SHN [1, 17–31] have been conducted. For instance, Le & Duy [1] predicted the αD half-lives of undetected transfermiums with $Z = 105 - 113$ based on barrier penetrability using the α-core interaction potentials, which are in terms of optical-model [32] and proximity potential [33], and semi-empirical Viola-Seaborg [22, 23], Royer [24, 25], and Akraway [26, 31] formulae. Ghodsi *et al* [18] calculated αD half-lives for SHN with $Z = 106 - 118$ using the proximity [34] and Ngo80 [35] potentials, the Viola-Seaborg, Royer, and universal decay law [36] formulae. In another work [17], Anghel *et al* employed the α-resonance [37], shell model [38], Brown formula [28], and experimental-data fitting approaches to investigate the half-lives of transfermiums with $Z = 104 - 112$. Considering the previous studies, a large uncertainty, up to 5 orders of magnitude, was found in predictions of αD half-lives. For SF studies, Xu *et al* [39] solved the multidimensional penetration problem in the quantum tunnelling effect using parabolic-potential approximation for SHN up to ^{286}114. The semi-empirical formula introduced in such work was in a good agreement with experimental data. Karpov *et al* [27] latter improved SF half-life predictions by fitting experimentaldata to known SHN with $100 \leqslant Z \leqslant 120$ and $240 \leqslant A \leqslant 310$, in which fission barrier and odd/even correction were taken into account. The formalism proposed by Karpov *et al* was then developed by updating new odd/even correction in [17]. In general, studies for SF are more complicated because of ambiguous fission mechanism and unavoidable uncertainties in parameters (i.e., nuclear mass, fission barrier, the number of evaporated neutrons, etc.) in SF half-life calculations [1, 39, 40], leading to a need of more studies on this issue.

At the present, predictions of SF and αD half-lives [1, 17, 18] are very uncertain due to lack of both measurements and calculations, especially for SHN beyond $Z = 118$. In addition, the island of stability is expected to be extended up to nuclei around the nuclear magic numbers $Z = 126$ and $N = 184$ [41–43]. Notice that the productions of the ^{309,312}126 nuclei seem be possible because of their relatively large synthesis cross sections [44]. Therefore, the half-lives and the decay mode of these isotopes are necessary to evaluate their observability in laboratories. In this paper, from the point of view of experimentalists, we identify the decay mode based on the SF and αD half-lives of $^{309-312}$126, which are in the vicinity of $N = 184$, and their daughters for future synthesis experiments. Because of small differences from measured data, as analyzed in previous studies [1, 45, 46], the Viola-Seaborg, Royer, Akrawy, Brown [28, 46], modified Royer [25, 47, 48], Qian [46, 49], and Ni [50, 51] semi-empirical formulae are utilized to calculate αD half-lives, whereas the Xu, Karpov, and Anghel empirical models are employed for estimating SF half-lives in our work.

The present paper is organized as follows. Theoretical framework for predicting SF and αD half-lives is described in section 2. The results of the half-lives and identification of decay mode are discussed in section 3. The summary of this study is given in section 4.

2. Theoretical framework

2.1. Spontaneous fission half-life

The SF half-lives of the investigated nuclei, $^{309-312}$126, are calculated using the phenomenological formula proposed by Xu *et al* [39] as

$$T_{SF} = \exp\left[2\pi\left(C_0 + C_1 A + C_2 Z^2 + C_3 Z^4 + C_4 (N - Z)^2 - 0.13323\frac{Z^2}{A^{1/3}} + 11.64\right)\right], \tag{1}$$

where $C_0 = -195.09227$, $C_1 = 3.10156$, $C_2 = -0.04386$, $C_3 = 1.40301 \times 10^{-6}$, and $C_4 = -0.03199$ are fitting coefficients, which are determined based on experimental data.

By considering the influence of the even/odd property and fission barrier on SF half-life, Karpov *et al* [27] developed a semi-empirical approach in terms of fissionability parameter ($\xi = Z^2/A$) as

$$\log_{10}(T_{SF}(s)) = 1146.44 - 75.3153\xi + 1.63792\xi^2 - 0.0119827\xi^3 + B_f(7.23613 - 0.0947022\xi) + h\,, \tag{2}$$

where B_f and h are the fission barrier (in MeV) and even/odd correction factor, respectively. The fission barrier values calculated by Koura *et al.* in [52] are utilized in the present study. The latter factor determined using experimental data and reliable calculations is given by [27]

$$h = 0\ (even - even),\ 1.538\,97\ (odd - A),\ \text{and}\ 0.80822\ (odd - odd)\,. \tag{3}$$

This factor has been reviewed by Anghel *et al* for specific even/odd isotopes. Thus, the updated values of the even/odd correction factor read [17]

$$h = 0\ (even - even),\ 2.007\ (even - odd),\ 2.822\ (odd - even),\ \text{and}\ 3.357\ (odd - odd)\,. \tag{4}$$

Obviously, the parameter sets from Karpov and Anghel formalisms are different from each other for even–odd, odd–even, and odd–odd nuclei.

2.2. Alpha decay half-life

Recently, many theoretical models have been developed for estimating αD half-life ($T_{1/2\alpha}$). Among them, semi-empirical approaches are good candidates for predicting the half-life since they are not only simple but also very efficient in use, especially for experimentalists. By considering results of previous works, we employ eight models, which are the most appropriate approaches to reproduce experimental data, to estimate the half-lives for isotopes in the α-decay chains of the $^{309-312}126$ nuclei.

2.2.1. Viola-Seaborg formula (VS)

One of early methods, namely Viola-Seaborg formula, is often used to estimate the half-lives of α decay, which is given by [22, 23]

$$\log_{10}(T^{VS}_{1/2\alpha}(s)) = (aZ + b)Q_\alpha^{-1/2} + cZ + d + f, \tag{5}$$

where the coefficients a, b, c, and d were determined by fitting experimental data to known SHN. Their updated values are $a = 1.64062$, $b = -8.54399$, $c = -0.19430$, $d = -33.90540$; and $f = 0, 0.5720, 0.8937$, and 0.9380 for even–even, odd–even, even–odd, and odd–odd nuclei, respectively [53]. Obviously, this relation is a function of the proton number (Z) and αD Q-value (Q_α). Notice that the recent updated Q_αs in the AME2016 database [54] are used to improve accuracy in this study.

2.2.2. Royer formula (R)

Royer developed a semi-empirical model for αD half-life, which depends on the released energy (Q_α) in the decay, atomic (Z) and mass (A) numbers of mother nuclei. Using this method, the half-life can be calculated by [24–26]

$$\log_{10}(T^{R}_{1/2\alpha}(s)) = a + bA^{1/6}\sqrt{Z} + \frac{cZ}{\sqrt{Q_\alpha}}. \tag{6}$$

The fitting parameters, which were updated in the recent study [26], for even–even, even–odd, odd–even and odd–odd isotopes are $a = -27.657, -28.408, -27.408$ and -24.763; $b = -0.966, -0.920, -1.038$, and -0.907; $c = 1.522, 1.519, 1.581$, and 1.410, respectively.

2.2.3. Akarawy approach (Akra.)

In a development, Akrawy *et al* considered the dependence of αD half-life on the isospin asymmetry $I = (N - Z)/A$, together with the even/odd property of nuclei, to proposed a novel relation, which reads [26, 31]

$$\log_{10}(T^{Akra.}_{1/2\alpha}(s)) = a + bA^{1/6}\sqrt{Z} + \frac{cZ}{\sqrt{Q_\alpha}} + dI + fI^2, \tag{7}$$

where the fitting parameters were determined using experimental data of 188 even–even, 147 even–odd, 131 odd–even, and 114 odd–odd α emitters, which gave $a = -27.8370, -28.2245, -26.8005$, and -23.6354; $b = -0.94199975, -0.8629, -1.10783$, and -0.891; $c = 1.5343, 1.537\,74, 1.5585$, and 1.404; $d = -5.7004, -21.145, 14.8525$, and -12.4255; and $f = 8.785, 53.890, -30.523$, and 36.9005, respectively [26].

2.2.4. universal scaling law of Brown method (SLB)

Another semi-empirical formula was suggested by Brown based on experimental data in 1992 for predicting αD half-life, which is described by [28, 46]

$$\log_{10}(T^{SLB}_{1/2\alpha}(s)) = aZ_D^{0.6}Q_\alpha^{-0.5} + b. \tag{8}$$

Since many new SHN have been observed up to date, the fitting coefficients (a, b) in equation (8) have been updated to improve the accuracy of half-life predictions. The values of these coefficients for even–even, even–odd, odd–even and odd–odd isotopes, respectively, are $a = 9.21067, 9.717\,86, 10.041\,41$, and $9.018\,62$; and $b = -49.58840, -51.60875, -53.45769$, and -47.88299 [46].

2.2.5. Modified Royer formula (mR1)

Since angular momentum (l) can be changed in the α decay, especially for mother and daughter nuclei whose spin-parities are different from each other, Royer suggested a new l-dependent function for αD half-life as [25, 47]

$$\log_{10}(T^{mR1}_{1/2\alpha}(s)) = a + bA^{1/6}\sqrt{Z} + \frac{cZ}{\sqrt{Q_\alpha}} + d\frac{ZNA[l(l+1)]^{1/4}}{Q_\alpha} + fA[1-(-1)^l]. \tag{9}$$

By fitting experimental data to 356 α nuclei, the coefficients in the equation above were deduced as $a = -25.31901, -27.87915, -28.64233$, and -28.61797; $b = -1.15847, -1.06904, -1.06471$, and -1.05756; $c = 1.58439, 1.611\,14, 1.631\,94$, and $1.631\,92$; $d = 0, 2.343\,01 \times 10^{-6}, 1.49479 \times 10^{-6}$, and 1.90030×10^{-6}; and

$f = 0, 0.000\,64, 0.001\,80$, and $0.001\,20$ for even–even, even–odd, odd–even, and odd–odd nuclei, respectively [47].

2.2.6. Modified Qian formula (mQ)

Taking both isospin asymmetry (I) and angular momentum (l) into account, Akwary *et al* modified a previous study, which proposed by Qian *et al* [49], to predict αD half-lives of SHN using a new empirical formula as [46]

$$\log_{10}(T^{mQ}_{1/2\alpha}(s)) = aZ_1Z_2\sqrt{\frac{\mu}{Q_\alpha}} + b\sqrt{Z_1Z_2\mu} + c\frac{l(l+1)A^{-1/6}}{\mu\sqrt{Z_1Z_2}} + d + fI + gI^2\,, \tag{10}$$

with $a = 0.41107, 0.442\,47, 0.446\,95$, and $0.433\,11$; $b = -1.44914, -1.41706, -1.31732$, and -1.40514; $c = 0, 5.258\,60, 4.947\,11$, and $4.388\,54$; $d = -14.87085, -16.75511, -21.24956$, and -17.14506; $f = 13.38618, -28.42224, -1.83758$, and -7.39768; and $g = -61.47107, 93.534\,85, -16.49410$, and $21.414\,28$ for even–even, even–odd, odd–even, and odd–odd nuclei, respectively. Similarly to the modified Royer formula ($T^{mR}_{1/2\alpha}$) [47], these values were determined based on experimental data of 356 α nuclei [46].

2.2.7. Modified Ni approach (mNi)

In another work, Akrawy *et al* also improved the αD half-life calculation based on a study of Ni *et al* [50] by adding isospin (I) and a term $[l(l+1)]$ of the centrifugal potential. The developed formula is given by [51]

$$\log_{10}(T^{mNi}_{1/2\alpha}(s)) = aZ_1Z_2\sqrt{\frac{\mu}{Q_\alpha}} + b\sqrt{Z_1Z_2\mu} + c + dI + fI^2 + g[l(l+1)]\,, \tag{11}$$

with $a = 0.41107, 0.441\,45, 0.446\,60$, and $0.433\,23$; $b = -1.44914, -1.42068, -1.32208$, and -1.40527; $c = -14.87085, -16.59713, -21.09761$, and -17.13866; $d = 13.38618, -27.68464, -1.64226$, and -7.66291; $f = -61.47107, 91.704\,05, -17.02692$, and $22.269\,25$; and $g = 0, 0.079\,47, 0.077\,67$, and $0.069\,02$ for even–even, even–odd, odd–even, and odd–odd nuclei, respectively.

2.2.8. Modified Royer approach taking isospin and angular momentum (mR2)

Recently, Deng *et al* took the centrifugal potential into the original formula of Royer model [equation (6)] to give a developed formula, which reads [48]

$$\log_{10}(T^{mR2}_{1/2\alpha}(s)) = a + bA^{1/6}\sqrt{Z} + \frac{cZ}{\sqrt{Q_\alpha}} + d[l(l+1)] + f\,, \tag{12}$$

with $a = -26.8125$, $b = -1.1255$; c=1.6057, d=0.0513, and $f = 0, 0.3625, 0.2812$, and 0.7486 for even–even, even–odd, odd–even and odd–odd isotopes, respectively [48].

The angular momentum l in the aforementioned formulae (equations (9)–(12)) can be determined using the conservation rules of momentum and parity as [55, 56]

$$\begin{aligned} l &= \Delta \quad \text{for even } \Delta \text{ and } \pi_M = \pi_D \\ &= \Delta + 1 \quad \text{for even } \Delta \text{ and } \pi_M \neq \pi_D \\ &= \Delta \quad \text{for odd } \Delta \text{ and } \pi_M \neq \pi_D \\ &= \Delta + 1 \quad \text{for odd } \Delta \text{ and } \pi_M = \pi_D, \end{aligned} \tag{13}$$

where $J_M - \pi_M$ and $J_D - \pi_D$ are the spin-parities of the mother and daughter nuclei, respectively; $\Delta = |J_M - J_D|$ is the difference between mother and daughter spins. Notice that, in the present study, spin-parities of the concerned nuclei are taken from the calculation of Moller *et al* [57].

As early mentioned, the de-excitation of super-heavy nuclei can mainly proceed through alpha emission or spontaneous fission. The competition between these processes is determined based on a branching ratio, B_{SF}, which is defined as

$$B_{SF} = \frac{T_{1/2\alpha}}{(T_{1/2\alpha} + T_{SF})} \times 100\,(\%)\,. \tag{14}$$

The spontaneous fission is defined as a dominant if B_{SF} exceeds 50%, and vice versa if B_{SF} is less than 50%. The α-decay branching can be determined as $B_\alpha = 100 - B_{SF}(\%)$.

3. Results and discussion

We systematically calculated the SF half-lives of the isotopes in the α-decay chains of the four unknown super-heavy nuclei, $^{309-312}126$, by using three models described in equations (1)–(4). Eight semi-empirical formulae expressed in equations (5)–(12) were utilized to estimate the αD half-lives of these isotopes. Since the SF half-life calculations based on the Anghel and Karpov formulae were different from each other due to the difference

Table 1. Spontaneous-fission half-lives (in seconds, logarithmic scale) of the nuclei in the α-decay chains of ^{309,311}126 calculated using the Anghel, Karpov, and Xu models. B_{SFA}^{AK} and B_{SFA}^{Xu} (or B_{SFB}^{AK} and B_{SFB}^{Xu}) denote the SF branching ratios (in %) estimated using SF half-lives (Anghel and Karpov models (AK) and Xu method) and α-decay half-lives in the dataset A (or dataset B).

Z	N	A	B_f(MeV)	Xu	Anghel	Karpov	AK average	B_{SFA}^{AK}	B_{SFA}^{Xu}	B_{SFB}^{AK}	B_{SFB}^{Xu}
126	183	309	7.788	42.084	−4.150	−4.618	−4.324	13.3	0.0	26.6	0.0
124	181	305	8.208	31.543	−0.751	−1.219	−0.925	0.0	0.0	0.0	0.0
122	179	301	8.168	22.608	1.222	0.754	1.048	0.0	0.0	0.0	0.0
120	177	297	7.700	15.187	1.795	1.327	1.622	0.0	0.0	0.0	0.0
118	175	293	6.667	9.189	0.539	0.071	0.365	0.3	0.0	1.5	0.0
116	173	289	5.173	4.527	−2.338	−2.806	−2.512	97.3	0.0	99.7	0.0
114	171	285	3.685	1.112	−5.538	−6.006	−5.712	100.0	4.1	100.0	2.7
112	169	281	2.152	−1.142	−9.149	−9.617	−9.323	100.0	76.6	100.0	99.3
110	167	277	1.915	−2.319	−8.961	−9.429	−9.134	100.0	56.7	100.0	97.3
108	165	273	2.661	−2.503	−5.556	−6.024	−5.729	100.0	99.7	100.0	100.0
106	163	269	3.437	−1.776	−1.734	−2.202	−1.908	100.0	100.0	100.0	100.0
104	161	265	4.089	−0.218	2.059	1.591	1.885	99.8	100.0	100.0	100.0
102	159	261	4.644	2.092	5.938	5.470	5.764	19.3	99.9	98.3	100.0
100	157	257	5.201	5.077	10.297	9.829	10.124	0.0	97.5	1.1	99.9
126	185	311	6.768	40.594	−5.751	−6.219	−5.925	93.7	0.0	97.9	0.0
124	183	307	8.253	30.026	0.101	−0.367	−0.073	0.0	0.0	0.0	0.0
122	181	303	8.381	21.064	2.411	1.943	2.237	0.0	0.0	0.0	0.0
120	179	299	8.237	13.616	3.778	3.310	3.604	0.0	0.0	0.0	0.0
118	177	295	7.193	7.591	2.456	1.988	2.282	0.0	0.0	0.0	0.0
116	175	291	6.044	2.902	0.535	0.067	0.361	13.2	0.0	61.1	0.5
114	173	287	4.512	−0.541	−2.773	−3.242	−2.947	100.0	95.4	100.0	95.2
112	171	283	2.764	−2.822	−6.995	−7.463	−7.169	100.0	100.0	100.0	100.0
110	169	279	1.717	−4.027	−9.262	−9.730	−9.436	100.0	100.0	100.0	100.0
108	167	275	2.337	−4.239	−6.207	−6.675	−6.381	100.0	100.0	100.0	100.0
106	165	271	3.046	−3.540	−2.544	−3.012	−2.718	100.0	100.0	100.0	100.0
104	163	267	3.704	−2.010	1.340	0.872	1.166	99.9	100.0	100.0	100.0
102	161	263	4.274	0.272	5.363	4.895	5.189	98.2	100.0	99.8	100.0
100	159	259	4.753	3.228	9.546	9.078	9.372	10.0	100.0	98.8	100.0

between their even–odd correction factors (h), we classified the investigated isotopes into odd-A and even–even groups corresponding to the ^{309,311}126 and ^{310,312}126 α-decay chains, respectively. Because of the remarkable difference among the results of the Xu model and those of the Anghel and Karpov methods, the branching ratios were separated into two groups, Xu (B_{Xu}) and AK (B_{AK}), corresponding to the Xu and the average values of the SF half-lives deduced using the Anghel and Karpov formalisms, respectively. By considering the dependence of formalisms on the angular momentum, we categorized the αD half-lives into set A (including the Viola-Seaborg (VS), Royer (R), Akrawy ($Akra.$), and Brown (SLB) methods) and set B (consisting of the modified formulae of Royer ($mR1$, $mR2$), Qian (mQ), and Ni (mNi) models). The branching ratios were considered based on the SF half-lives and average values of the αD half-lives of each dataset.

Tables 1 and 2 present the SF half-lives (in seconds, logarithmic scale) of the isotopes in the four decay chains of interest. The results show that the SF half-lives based on the Anghel and Karpov methods are similar to each other for the even–even nuclei in the ^{310,312}126 chains due to the same values of the even/odd correction parameters, $h = 0$. In contrast, the SF half-lives deduced using the Anghel model are a factor of about 3 higher than those obtained by using the Karpov formula for the even–odd nuclei in the ^{309,311}126 chains. However, the half-lives of these models are closer to each other, but different from those of the Xu approach. By considering average values of the Anghel and Karpov SF half-lives, minima are observed at $Z = 110$ (figure 1(A)), but at $Z = 108$ in calculations using the Xu method (figure 1(B)). Moreover, the dependence of SF half-lives on atomic numbers is totally inversely changed at $Z = 110$ (Ds) and $Z = 120$, as can be seen in figure 1(A). The SF half-lives are almost linearly decreased with the increasing atomic numbers of $Z = 100 - 110$, while it seemly follows a parabolic function with maxima at $Z = 120$ for $Z = 112 - 126$ nuclei.

The dependence of the Anghel and Karpov SF half-lives on atomic numbers can be understood by the impacts of shell effects on the SF half-lives through fission barriers. Notice that the fission barrier strongly depends the shell corrections [59–61]. As shown in figure 1(C), maximum (or minimum) values of shell correction can be achieved in the vicinity of $Z = 110$ (or $Z = 116$), leading to minimum (or maximum) of the SF half-lives. The minima around $Z = 110$are consistent with the study on fission barrier height in [52], which have predicted a basin region existing around $Z = 110$ nuclei (i.e., ^{278}Ds) for fission barriers. Besides, the SF half-lives are similar to the fission barriers (as shown in the inset of figure 1(A)) in the trend of response to atomic

Table 2. Spontaneous-fission half-lives (in seconds, logarithmic scale) of even–even nuclei in the α-decay chains of ^{310,312}126 calculated using the Anghel, Karpov, and Xu models. B^{AK}_{SFA} and B^{Xu}_{SFA} (or B^{AK}_{SFB} and B^{Xu}_{SFB}) denote the SF branching ratios (in %) estimated using SF half-lives (Anghel and Karpov models (AK) and Xu method) and α-decay half-lives in the dataset A (or dataset B).

Z	N	A	B_f	Xu	Anghel	Karpov	AK Average	B^{AK}_{SFA}	B^{Xu}_{SFA}	B^{AK}_{SFB}	B^{Xu}_{SFB}
126	184	310	7.873	41.428	−5.522	−5.522	−5.522	45.9	0.0	7.7	0.0
124	182	306	8.228	30.874	−2.334	−2.334	−2.334	0.2	0.0	0.0	0.0
122	180	302	8.333	21.925	−0.039	−0.039	−0.039	0.0	0.0	0.0	0.0
120	178	298	8.197	14.490	1.387	1.387	1.387	0.0	0.0	0.0	0.0
118	176	294	6.942	8.480	−0.477	−0.477	−0.477	0.6	0.0	0.1	0.0
116	174	290	5.608	3.804	−2.917	−2.917	−2.917	97.4	0.0	90.3	0.0
114	172	286	4.046	0.375	−6.323	−6.323	−6.323	100.0	14.6	100.0	4.7
112	170	282	2.380	−1.893	−10.319	−10.319	−10.319	100.0	95.9	100.0	87.8
110	168	278	1.775	−3.084	−11.246	−11.246	−11.246	100.0	93.4	100.0	79.6
108	166	274	2.577	−3.282	−7.638	−7.638	−7.638	100.0	99.9	100.0	99.7
106	164	270	3.291	−2.569	−3.981	−3.981	−3.981	100.0	100.0	100.0	99.9
104	162	266	3.885	−1.025	−0.346	−0.346	−0.346	100.0	100.0	100.0	100.0
102	160	262	4.532	1.271	3.896	3.896	3.896	95.5	100.0	94.2	100.0
100	158	258	4.963	4.242	7.865	7.865	7.865	9.1	99.8	8.7	99.8
126	186	312	5.705	39.581	−9.938	−9.938	−9.938	100.0	0.0	100.0	0.0
124	184	308	8.322	29.000	−1.380	−1.380	−1.380	0.0	0.0	0.0	0.0
122	182	304	8.421	20.024	0.817	0.817	0.817	0.0	0.0	0.0	0.0
120	180	300	8.237	12.563	2.043	2.043	2.043	0.0	0.0	0.0	0.0
118	178	296	7.625	6.525	1.883	1.883	1.883	0.0	0.0	0.0	0.0
116	176	292	6.508	1.822	0.062	0.062	0.062	11.8	0.2	3.2	0.1
114	174	288	5.038	−1.635	−3.044	−3.044	−3.044	100.0	99.0	99.9	96.9
112	172	284	3.187	−3.930	−7.554	−7.554	−7.554	100.0	100.0	100.0	100.0
110	170	280	1.853	−5.149	−10.682	−10.682	−10.682	100.0	100.0	100.0	100.0
108	168	276	2.068	−5.375	−8.888	−8.888	−8.888	100.0	100.0	100.0	100.0
106	166	272	2.889	−4.689	−4.828	−4.828	−4.828	100.0	100.0	100.0	100.0
104	164	268	3.541	−3.173	−0.924	−0.924	−0.924	100.0	100.0	100.0	100.0
102	162	264	4.113	−0.905	3.156	3.156	3.156	100.0	100.0	100.0	100.0
100	160	260	4.601	2.037	7.425	7.425	7.425	92.5	100.0	92.8	100.0

numbers. On the other hand, the influence of the shell effects is also exhibited in calculations using the Xu model. It is clear in figure 1(B) that the large shell correction can result in the minimum values of the Xu SF half-lives around $Z = 108 - 110$. The results also reflect that SF half-lives are very sensitive to shell correction (and, subsequently, fission barrier). This sensitivity is consistent with the results in previous works [27, 62, 63]. Hence, the accuracy of shell effects and fission barrier is important to investigate the spontaneous fission.

It is surprisely found that there is a very large dicrepancy, up to 50 orders of magnitude, between the SF half-lives determined using the Xu model and those of the Anghel and Karpov approaches, as can be seen in figure 1(D). The deviation within 10 orders of magnitude is observed for the nuclei with the atomic numbers of $Z = 100 - 120$, but it is from 10 to 50 orders of magnitude for the heavier isotopes with $Z \geqslant 120$. The large discrepancy can be explained by the difference in the theories employed to established the formulae. For instance, the inverted parabolic potential was mainly considered as the nuclear interaction potential in the Xu model [39], while the barrier height on potential energy surface was mainly taken into account in the Anghel and Karpov methods [27, 64]. The interaction potential was also modified by taking the attractive strong forces between the nucleons, asymmetric/symetric charge distributions, and isospin effect into account in the Xu model [39]. In addition, the fitting parameters in the Xu model were deduced based on experimental data of only 45 even–even nuclei from ^{232}Th to ^{286}114, whereas fitting coefficients in the Anghel and Karpov formulae were obtained based on all the existing measured data together with reliable theoretical calculations in a wide range of isotopes up to $Z = 120$ and $N = 190$ [17, 27]. Obviously, the results in this study indicate that studies on fission mechanism and microscopic effects in SHN are the main way to narrow the large uncertainty in the SF half-life.

For α decay, the half-lives ($T_{1/2\alpha}$) of isotopes in the decay chains of the $^{309-312}$126 nuclei calculated using eight aforementioned methods are presented in tables 3, 4. The half-lives are from a few microseconds (10^{-6} s) to a few thousands of years (10^{10} s) for the isotopes of interest. The isotopes with $Z = 124$ and 126 have shortest αD half-lives while the $Z = 100$, 102 nuclei are the most stable nuclei in the decay chains. Additionally, the half-lives drastically increase by the dreasing atomic numbers with an average rate of one order of magnitude per one emitted alpha. For syntheses of SHN, the half-lives play a key role for a sucessful observation of new isotopes. Since the typical separation time of separators for identifying SHN at recent accelerator facilities is about 1–2 μs [16], in practical, it is difficult to detect directly the $^{309-312}$126 nuclei in experiments because of their

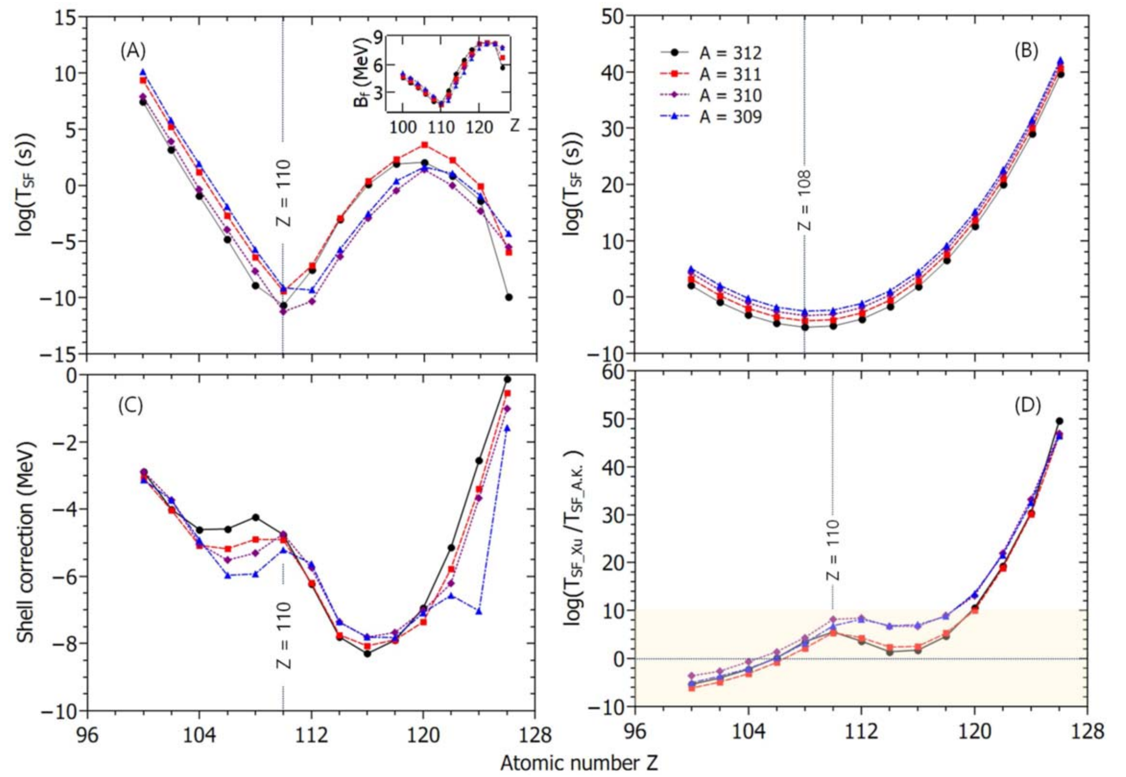

Figure 1. Spontaneous-fission half-lives (in seconds, logarithmic scale) of even–odd nuclei in four α-decay chains of the unknown $^{309-312}126$ nuclei calculated using the Anghel and Karpov formulae (A), and the Xu model (B). Notice that panel (A) shows the average values of T_{SF} based on the two formulae. The shell corrections obtained from [58] are shown in panel (C). The average SF half-lives calculated using the Anghel and Karpov methods are compared to those based on the Xu model in panel (D).

short half-lives. Fortunately, with longer half-lives of a few milliseconds, their descendants (i.e., $^{289-292}116$) can be alternatives for observations as evidences for existence of new SHN, $^{309-312}126$.

As can be seen in figure 2, the half-lives calculated using *VS*, *R*, *Akra.* , and *SLB* methods (dataset A) are close to each other, especially for isotopes having neutron numbers $N < 180$. The differences among them are only up to one order of magnitude. The *Akra.* approach slightly dominates the others while the *SLB* one generates smallest values of the half-lives. On the other hand, the half-lives of the *mR*1 (or *mR*2) are longest (or shortest) compared to those of the *mQ*, *mNi*, and *mR*2 (or *mR*1) in the dataset B. The values of these calculations differ about 1-2 orders of magnitude from those of the others. By considering two datasets, we found that the half-lives of all the methods are mostly the same for the even–even nuclei. However, the half-lives in the dataset B are about 1-2 orders of magnitude longer than those in the dataset A for even–odd isotopes. This difference can be understood by the dependence of the formalisms on the angular momenta in the α decay. Since the angular momenta are significantly affected by spin-parities of mother and daughter nuclei, the change in spin-parities of even–odd nuclei in the αD chains of $^{309,311}126$ strongly results in the half-lives of the dataset B. In contrast, there is no change in spin-parities in the α emission of the even–even isotopes ($^{310,312}126$), leading to $l = 0$ which removes the angular-momentum dependence in the calculations of the dataset B. Subsequently, the half-lives in two datasets are mostly similar to each other for even–even nuclei. Regardless the momentum, in general, the employed semi-empirical formulae are consistent with each other because they have been calibrated using experimental data for their coefficients. Hitherto, since the spin-parities of SHN are very uncertain, reliable calculations and/or precise spin-parity measurements for SHN are necessary to reduce the uncertainty in αD half-life predictions.

On the other hand, the results indicate that longer αD half-lives are observed for the $^{263,264}No_{161,162}$, $^{265,266}Rf_{161,162}$, $^{283,284}Cn_{171,172}$, and $^{285-287}Fl_{171\text{-}173}$ isotopes with neutron numbers in the vicinity of $N = 162$, 172 where the smallest Q-values are observed, as can be seen in the insets of panels (A) - (D). These longer αD half-lives can be explained by the enhanced stability due to the deformed shell closures at or in the vicinity of $N = 162$, 172 [9, 45, 65]. Notice that the step down of the two-neutron separation energies [54], as an additional evidence for shell closures, is also found in these isotopes. Moreover, the trend of half-lives strongly depends on Q-values rather than on the angular momenta in the α decay. Subsequently, the more stability of a super-heavy nuclei can be predicted by considering the Q-values instead of the angular momenta. Unfortunately, theoretical calculations are the main approach to determine the αD Q-values, which are very uncertain at present. Hence,

Table 3. α-decay half-lives (in seconds, logarithmic scale) of isotopes in the α-decay chains of the unknown ^{309,311}126 nuclei. $B^{SetA}_{\alpha-AK}$, $B^{SetA}_{\alpha-Xu}$ (or $B^{SetB}_{\alpha-AK}$, $B^{SetB}_{\alpha-Xu}$) are branching ratios (in %) of α-decay mode estimated using average α-decay half-lives of set A (or set B), SF half-lives of Xu method, and average SF half-lives of Anghel and Karpov models.

Z	N	A	Q_α (MeV)	J^π	l	VS	R	Akra.	SLB	mR1	mQ	mNi	mR2	Set A	Set B	$B^{SetA}_{\alpha-AK}$	$B^{SetA}_{\alpha-Xu}$	$B^{SetB}_{\alpha-AK}$	$B^{SetB}_{\alpha-Xu}$
126	183	309	14.591	3/2+	1	−5.61	−5.15	−4.75	−5.73	−4.18	−6.01	−5.93	−6.23	−5.14	−4.76	86.7	100.0	73.4	100.0
124	181	305	14.035	1/2—	1	−5.08	−4.71	−4.33	−5.29	−3.69	−5.46	−5.38	−5.72	−4.70	−4.27	100.0	100.0	100.0	100.0
122	179	301	13.474	1/2+	0	−4.52	−4.23	−3.86	−4.80	−4.90	−4.92	−4.93	−5.27	−4.22	−4.98	100.0	100.0	100.0	100.0
120	177	297	12.907	1/2+	0	−3.91	−3.70	−3.35	−4.26	−4.31	−4.27	−4.28	−4.66	−3.69	−4.36	100.0	100.0	100.0	100.0
118	175	293	11.920	1/2+	2	−2.34	−2.25	−1.90	−2.84	−0.88	−2.36	−2.10	−2.77	−2.22	−1.44	99.7	100.0	98.5	100.0
116	173	289	11.100	5/2+	2	−0.99	−1.00	−0.66	−1.60	0.53	−0.87	−0.62	−1.40	−0.95	−0.02	2.7	100.0	0.3	100.0
114	171	285	10.560	3/2+	0	−0.24	−0.32	0.01	−0.88	−0.64	−0.25	−0.26	−0.95	−0.26	−0.44	0.0	95.9	0.0	97.3
112	169	281	10.450	3/2+	4	−0.57	−0.70	−0.39	−1.16	1.51	0.09	0.96	−0.28	−0.63	1.03	0.0	23.4	0.0	0.7
110	167	277	10.830	11/2+	4	−2.14	−2.27	−2.00	−2.60	−0.32	−1.62	−0.75	−1.89	−2.20	−0.76	0.0	43.3	0.0	2.7
108	165	273	9.700	3/2+	5	0.15	−0.09	0.19	−0.40	2.79	1.30	2.59	0.98	0.02	2.42	0.0	0.3	0.0	0.0
106	163	269	8.650	13/2—	5	2.62	2.27	2.56	2.01	5.51	4.06	5.34	3.52	2.43	5.15	0.0	0.0	0.0	0.0
104	161	265	7.810	3/2+	2	4.78	4.34	4.64	4.16	6.53	5.62	5.85	4.52	4.55	6.06	0.2	0.0	0.0	0.0
102	159	261	7.440	1/2+	4	5.39	4.91	5.20	4.86	7.89	6.86	7.69	5.89	5.14	7.53	80.7	0.1	1.7	0.0
100	157	257	6.864	9/2+	1	6.92	6.37	6.67	6.47	8.63	7.94	8.00	6.56	6.66	8.19	100.0	2.5	98.9	0.1
126	185	311	14.354	3/2+	2	−5.19	−4.77	−4.37	−5.36	−3.68	−5.38	−5.11	−5.63	−4.75	−4.25	6.3	100.0	2.1	100.0
124	183	307	13.794	1/2+	2	−4.63	−4.30	−3.91	−4.88	−3.16	−4.79	−4.52	−5.08	−4.28	−3.72	100.0	100.0	100.0	100.0
122	181	303	13.228	3/2+	2	−4.03	−3.79	−3.42	−4.37	−2.58	−4.15	−3.89	−4.50	−3.77	−3.15	100.0	100.0	100.0	100.0
120	179	299	12.657	1/2+	0	−3.39	−3.23	−2.87	−3.80	−3.82	−3.66	−3.67	−4.17	−3.21	−3.79	100.0	100.0	100.0	100.0
118	177	295	11.700	1/2+	0	−1.84	−1.79	−1.43	−2.39	−2.26	−1.96	−1.97	−2.60	−1.75	−2.13	100.0	100.0	100.0	100.0
116	175	291	10.890	1/2+	2	−0.47	−0.52	−0.17	−1.12	1.10	−0.25	0.01	−0.90	−0.46	0.56	86.8	100.0	38.9	99.5
114	173	287	10.160	3/2+	0	0.83	0.69	1.04	0.11	0.43	0.99	0.97	0.11	0.78	0.76	0.0	4.6	0.0	4.8
112	171	283	9.940	3/2+	7	0.80	0.61	0.94	0.12	4.22	2.92	5.39	2.94	0.71	4.82	0.0	0.0	0.0	0.0
110	169	279	10.080	15/2—	7	−0.23	−0.44	−0.14	−0.81	2.94	1.82	4.28	1.88	−0.34	3.70	0.0	0.0	0.0	0.0
108	167	275	9.440	3/2+	0	0.89	0.61	0.90	0.30	0.42	1.10	1.09	0.16	0.74	0.86	0.0	0.0	0.0	0.0
106	165	271	8.890	3/2+	5	1.85	1.50	1.79	1.28	4.67	3.29	4.57	2.70	1.66	4.33	0.0	0.0	0.0	0.0
104	163	267	7.890	13/2—	5	4.48	4.02	4.34	3.88	7.60	6.25	7.51	5.41	4.25	7.27	0.1	0.0	0.0	0.0
102	161	263	7.000	3/2+	0	7.19	6.63	6.97	6.60	6.91	8.19	8.16	6.68	6.92	7.89	1.8	0.0	0.2	0.0
100	159	259	6.470	1/2+	4	8.70	8.08	8.42	8.21	11.62	10.67	11.48	9.29	8.42	11.29	90.0	0.0	1.2	0.0

Table 4. α-decay half-lives (in seconds, logarithmic scale) of isotopes in the α-decay chains of the unknown ^{310,312}126 nuclei. $B^{SetA}_{\alpha-AK}$, $B^{SetA}_{\alpha-Xu}$ (or $B^{SetB}_{\alpha-AK}$, $B^{SetB}_{\alpha-Xu}$) are branching ratios (in %) of α-decay mode estimated using average α-decay half-lives of set A (or set B), SF half-lives of Xu method, and average SF half-lives of Anghel and Karpov models. Notice that spin-parities $J^{\pi} = 0^{+}$ and, subsequently, angular momenta $l = 0$ for all the even–even isotopes.

Z	N	A	Q_α (MeV)	VS	R	Akra.	SLB	mR1	mQ	mNi	mR2	Set A	Set B	$B^{SetA}_{\alpha-AK}$	$B^{SetA}_{\alpha-Xu}$	$B^{SetB}_{\alpha-AK}$	$B^{SetB}_{\alpha-Xu}$
126	184	310	14.476	−6.30	−5.46	−5.29	−5.93	−6.68	−6.62	−6.62	−6.50	−5.59	−6.60	54.1	100.0	92.3	100.0
124	182	306	13.918	−5.76	−4.99	−4.84	−5.50	−6.14	−6.10	−6.10	−5.98	−5.13	−6.08	99.8	100.0	100.0	100.0
122	180	302	13.355	−5.18	−4.48	−4.34	−5.03	−5.57	−5.53	−5.53	−5.41	−4.62	−5.51	100.0	100.0	100.0	100.0
120	178	298	12.786	−4.55	−3.93	−3.79	−4.50	−4.95	−4.92	−4.92	−4.79	−4.07	−4.89	100.0	100.0	100.0	100.0
118	176	294	11.840	−3.05	−2.52	−2.39	−3.21	−3.44	−3.41	−3.41	−3.27	−2.67	−3.38	99.4	100.0	99.9	100.0
116	174	290	11.000	−1.64	−1.19	−1.07	−1.97	−2.01	−1.99	−1.99	−1.84	−1.34	−1.95	2.6	100.0	9.7	100.0
114	172	286	10.370	−0.63	−0.25	−0.13	−1.07	−0.98	−0.97	−0.97	−0.81	−0.39	−0.93	0.0	85.4	0.0	95.3
112	170	282	10.170	−0.73	−0.38	−0.28	−1.12	−1.07	−1.09	−1.09	−0.92	−0.52	−1.04	0.0	4.1	0.0	12.2
110	168	278	10.470	−2.15	−1.80	−1.72	−2.34	−2.50	−2.56	−2.56	−2.38	−1.94	−2.49	0.0	6.6	0.0	20.4
108	166	274	9.570	−0.38	−0.11	−0.03	−0.72	−0.69	−0.75	−0.75	−0.56	−0.24	−0.68	0.0	0.1	0.0	0.3
106	164	270	8.990	0.65	0.87	0.93	0.26	0.37	0.29	0.29	0.49	0.74	0.37	0.0	0.0	0.0	0.1
104	162	266	7.550	4.87	4.97	5.05	4.17	4.69	4.62	4.62	4.85	4.87	4.71	0.0	0.0	0.0	0.0
102	160	262	7.250	5.25	5.32	5.39	4.63	5.10	5.01	5.01	5.26	5.23	5.11	4.5	0.0	5.8	0.0
100	158	258	6.660	6.93	6.95	7.01	6.30	6.85	6.73	6.73	7.01	6.87	6.84	90.9	0.2	91.3	0.2
126	186	312	14.238	−5.87	−5.07	−4.91	−5.57	−6.28	−6.23	−6.23	−6.10	−5.21	−6.20	0.0	100.0	0.0	100.0
124	184	308	13.675	−5.30	−4.58	−4.43	−5.11	−5.72	−5.68	−5.68	−5.54	−4.72	−5.65	100.0	100.0	100.0	100.0
122	182	304	13.108	−4.69	−4.04	−3.90	−4.61	−5.11	−5.09	−5.09	−4.94	−4.18	−5.05	100.0	100.0	100.0	100.0
120	180	300	12.534	−4.03	−3.45	−3.32	−4.05	−4.45	−4.44	−4.44	−4.29	−3.59	−4.40	100.0	100.0	100.0	100.0
118	178	296	11.955	−3.31	−2.80	−2.69	−3.44	−3.73	−3.74	−3.74	−3.58	−2.95	−3.69	100.0	100.0	100.0	100.0
116	176	292	10.774	−1.07	−0.67	−0.55	−1.48	−1.46	−1.46	−1.46	−1.29	−0.81	−1.41	88.2	99.8	96.8	99.9
114	174	288	10.072	0.18	0.51	0.62	−0.35	−0.19	−0.20	−0.20	−0.02	0.37	−0.15	0.0	1.0	0.1	3.1
112	172	284	9.600	0.88	1.15	1.25	0.30	0.52	0.49	0.49	0.69	1.02	0.56	0.0	0.0	0.0	0.0
110	170	280	9.810	−0.39	−0.12	−0.04	−0.78	−0.75	−0.82	−0.82	−0.61	−0.25	−0.74	0.0	0.0	0.0	0.0
108	168	276	9.280	0.47	0.69	0.75	0.04	0.13	0.04	0.04	0.27	0.56	0.13	0.0	0.0	0.0	0.0
106	166	272	8.680	1.63	1.79	1.85	1.14	1.33	1.21	1.21	1.46	1.67	1.32	0.0	0.0	0.0	0.0
104	164	268	8.040	3.05	3.15	3.21	2.51	2.80	2.67	2.67	2.94	3.05	2.78	0.0	0.0	0.0	0.0
102	162	264	6.820	7.08	7.08	7.15	6.31	6.93	6.80	6.80	7.11	7.00	6.93	0.0	0.0	0.0	0.0
100	160	260	6.300	8.62	8.58	8.65	7.87	8.54	8.39	8.39	8.73	8.52	8.53	7.5	0.0	7.2	0.0

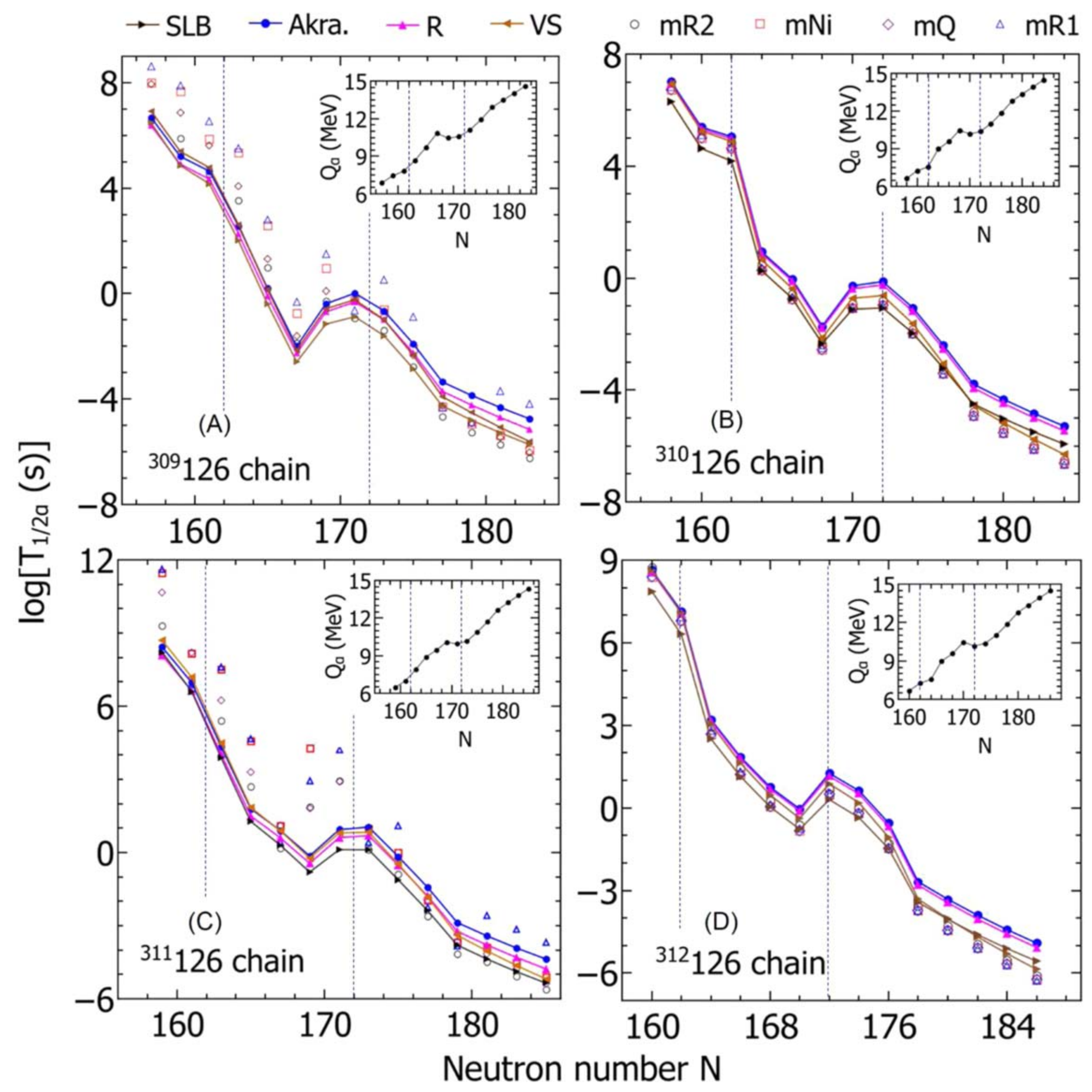

Figure 2. Comparisons of half-lives calculated using different methods for the decay chains of $^{309-312}126$. Insets show dependence of Q_αs on neutron numbers of the investigated isotopes. Dashed lines are to guide the eyes.

precise mass measurements for known super-heavy nuclei are encouraged to improve the accuracy of Q-value calculations and, subsequently, the αD half-lives.

A comparison for the calculated and measured αD half-lives is shown in figure 3. It is found that most of predicted half-lives in the dataset A (or dataset B) of 14 observed SHN in the αD chains of $^{309-312}126$ differ by 1-2 (or 1-4.5) orders of magnitude from the measured data [66–68], as can be seen in the right panel. This discrepancy is close to that observed in a systematic analysis for αD half-life predictions in [45]. It should be noted that the experimental data were also observed with an uncertainty of about one order of magnitude, as shown in the left panel. These results indicate that the *VS*, *R*, *Akra.*, and *SLB* methods (set A) with recent updated coefficients are reliable for predicting αD half-lives of SHN. On the other hand, the large difference between the dataset B and experimental data should be narrowed by improving accuracy of spin-parities of mother and daughter nuclei for more precise angular momenta in the approaches described in equations (9)–(12).

To determine the dominant in the competition between αD and SF, the average $T_{1/2\alpha}$s of the dataset A and of the dataset B are compared to the average SF half-lives (T_{SF}^{AK}) estimated using the Anghel and Karpov methods and to those determined based on the Xu model (T_{SF}^{Xu}), as can be seen in the left and right panels in figure 4, respectively. It is found that the difference between $T_{1/2\alpha}$s and T_{SF}^{AK} s is mostly smaller than 10 orders of magnitude for all the isotopes of interest [panels (A), (C)], but for only isotopes lighter than ^{294}Og ($Z = 118$) in the case of T_{SF}^{Xu} [panels (B), (D)]. A large difference, from 10 to 50 orders of magnitude, is observed for isotopes beyond ^{294}Og in the comparison to T_{SF}^{Xu}. Notice that there are exceptions for the ^{279}Ds ($Z = 110$) and ^{283}Cn ($Z = 112$) isotopes in the $^{311}126$ chain for the case of the dataset B [panel (C)]. The difference between the αD and SF half-lives of these exceptions is observed to be more than 10 orders of magnitude. This result is

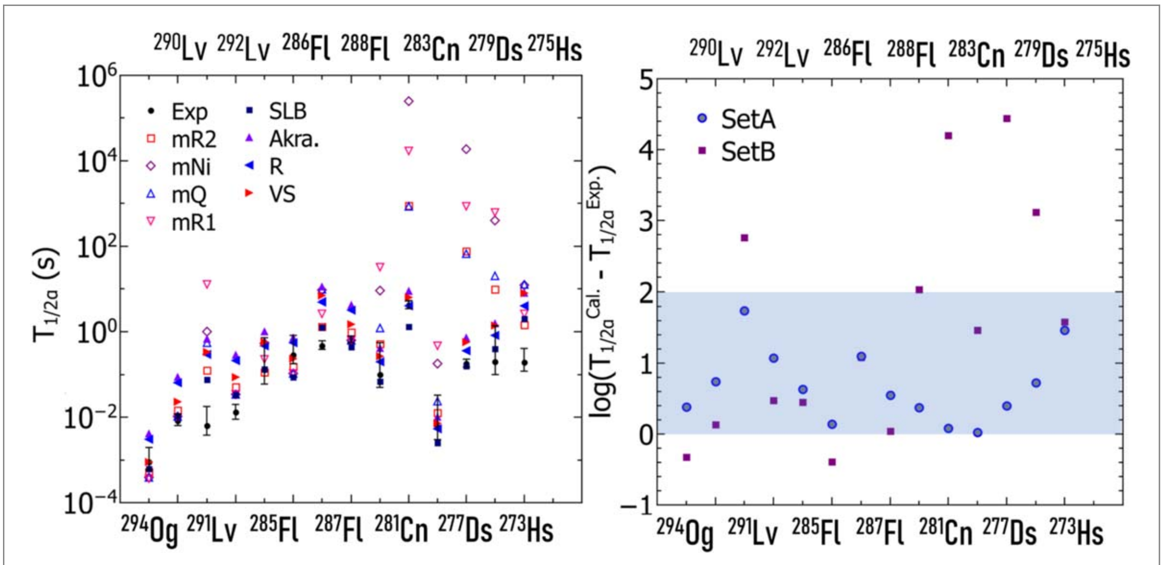

Figure 3. [Left panel] Average values of calculated αD half-lives (red squares) and available experimental data (black dots) [66–68] for 14 observed isotopes with $Z = 108 - 118$ in the αD chains of the $^{309-312}$126 isotopes. [Right panel] Ratios of the calculated to measured αD half-lives. Shaded area indicates a difference within two orders of magnitude among half-lives of set A and those of measurements.

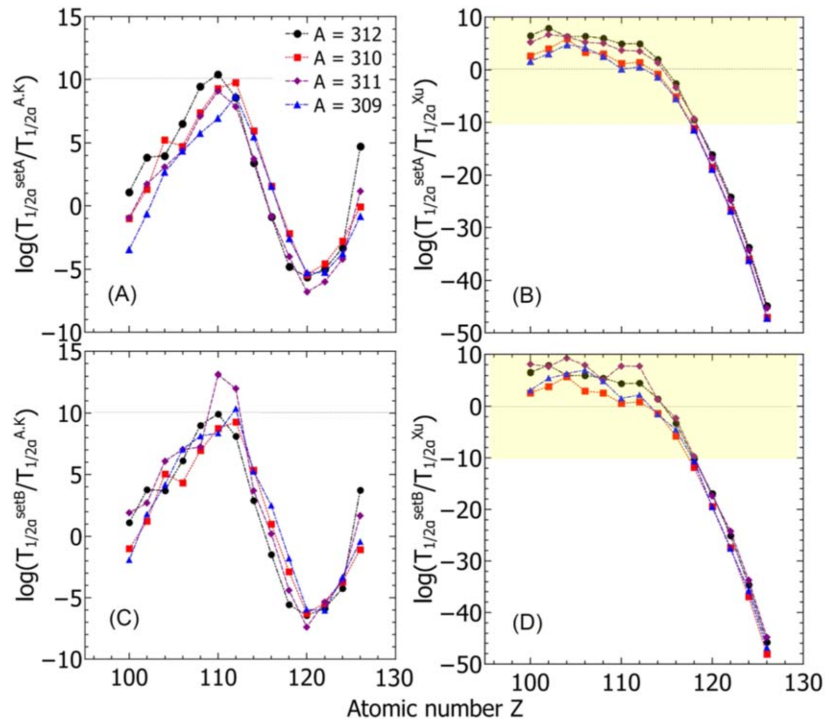

Figure 4. Differences between the SF and αD half-lives of super-heavy nuclei with $Z = 100 - 126$ in four αD chains of the $^{309-312}$126 isotopes when [panels (A), (C)] the Anghel and Karpov methods, and [panels (B), (D)] the Xu model were taken into account. Shaded areas indicate differences less than 10 orders of magnitude.

understood by the enhanced $T_{1/2\alpha}$s due to the larger angular momenta ($l = 7\hbar$) in the α decay of these isotopes compared to those of the others. On the other hand, it is also observed that the SF process dominates the α decay for nuclei with $Z = 102 - 114$, and vice versa for $Z = 116 - 126$ isotopes when the Anghel and Karpov methods are taken into account [panels (A), (C)]. However, by considering T_{SF}^{Xu}s, the dominance of the SF process is found for the $Z = 100 - 112$ nuclei, and vice versa for the $Z \geqslant 112$ ones [panels (B), (D)].

The branching ratios of the decay mode are estimated using equation (14). The values of B_{SF} (or B_α) based on the average values of T_{SF}^{AK}s, T_{SF}^{Xu}s, $T_{1/2\alpha}^{SetA}$ s, and $T_{1/2\alpha}^{SetB}$s are presented in the last four columns in tables 1 and 2 [or tables 3 and 4]. These results suggest that most of SHN with $Z \geqslant 118$ (or $Z = 102 - 112$) are classified as the αD (or SF) isotopes regardless any models, except for $^{311,312}126$. Together with these exceptions, arguments among the models are found for $^{285,286}114$ (Fl) and $^{289,290}116$ (Lv). In these cases, the SF process dominates the α decay if the Anghel and Karpov methods are taken into account, and vice versa if the Xu approach is employed. Discrepancies in decay mode determination also occur for $^{258,259}100$ (Fm) and $^{261}102$ (No), which have the αD mode when $T^{SetA}_{1/2\alpha}$ and T^{AK}_{SF} are considered. Besides, $^{257}100$ (Fm) can be an α-emitter (or SF isotope) for the calculation using $T^{SetA}_{1/2\alpha}$ (or $T^{SetB}_{1/2\alpha}$). On the other hand, the results indicate that 6α (or 9α) chain can be observed from $^{309,310}126$ if the dataset A (or B) is taken into account. For all the methods, the $^{311}126$ and $^{308}124$ can decay via a 6α and 5α chains, respectively. The α decay is not possible for the $^{312}126$ isotope if T^{AK}_{SF} is used in the estimation, but it decays via a 7α chain when T^{Xu}_{SF} is employed. Obviously, there are large discrepancies in the determination of decay mode. This result is understood by both differences among the models and uncertainties in calculating parameters such as Q-value, fission barrier, fitting coefficients, spin-parities, etc. Subsequently, more measurements for these parameters are highly demanded. Furthermore, both αD and SF half-lives of SHN must be precisely obtained at the same time to determine precisely the decay mode of SHN.

4. Conclusion

In this study, we evaluated α-decay and spontaneous-fission half-lives of 56 super-heavy nuclei in the α-decay chains of the unknown $^{309-312}126$ nuclei. The results indicate that fission barrier strongly impacts SF half-life while α-decay half-life is sensitive to Q-value and angular momentum in the emission, leading to uncertainty in half-lives of SHN because of poor precision of these quantities. Besides, most of calculated α-decay half-lives differ by about 1-2 orders of magnitude from the available experimental data. The results show that the semi-empirical approaches proposed by Viola-Seaborg, Royer, Akrawy and Brown are reliable to predict α-decay half-lives of SHN whereas the modified Royer, Ni, and Qian methods require more precise angular momentum to reproduce experimental data. On the other hand, we found that the SF half-lives based on the Anghel and Karpov approaches are almost similar to each other, but largely different from those of the Xu model, especially for $Z > 120$ isotopes. The large discrepancy among these results and uncertainties in the α-decay half-lives lead to the uncertainty in determination of decay mode for the isotopes of interest. For the α-decay, the results determined using the semi-empirical formulae are almost similar to each other in the same set of methods (set A or set B). By taking the average Anghel and Karpov SF half-lives together with average α-decay ones, we found that the decays of $^{309-311}126$ can initiate with the α emissions while $^{312}126$ starts with the spontaneous fission. In contrast, the $^{312}126$ isotope is identified as an α emitter if the Xu model is used for calculations. As a result, to detect the unknown $^{309-311}126$ nuclei, alpha spectrometers are strongly suggested to be employed in experiments for their observations. Finally, the results in this work provide useful information for further studies on SHN properties and productions.

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References

[1] Le N N and Duy N N 2020 Examination of α-decay half-lives of undetected transfermium isotopes *Int. J. Mod. Phys.* E (https://doi.org/10.1142/S0218301320500858)
[2] Delion D S, Liotta R J and Wyss R 2007 *Phys. Rev.* C **76** 044301
[3] Samanta C, Basu D N and Chowdhury P R 2007 *J. Phys. Soc. Japan.* **76** 124201
[4] Zhang H F and Royer G 2007 *Phys. Rev.* C **76** 047304
[5] Poenaru D N, Greiner W and Depta K 1986 *At. Data Nucl. Data Tables* **34** 423
[6] Poenaru D N, Plonski I H and Greiner W 2006 *Phys. Rev.* C **74** 014312

[7] Bohr N and Wheeler J A 1939 *Phys. Rev.* **56** 426
[8] Audi G, Bersillon O, Blachot J and Wapstra A H 2003 *Nucl. Phys.* A **729** 3
[9] Hofmann S and Munzenberg G 2000 *Rev. Mod. Phys.* **72** 733
[10] Oganessian Y T *et al* 2005 *Phys. Rev.* C **72** 034611
[11] Ginter T N *et al* 2003 *Phys. Rev.* C **67** 064609
[12] Turler A *et al* 2003 *Eur. Phys. J.* A **17** 505
[13] Gan Z G *et al* 2004 *Eur. Phys. J.* A **20** 385
[14] Morita K *et al* 2007 *J. Phys. Soc. Jpn.* **76** 043201
[15] Folden C M III *et al* 2004 *Phys. Rev. Lett.* **93** 212702
[16] Hofmann S 2011 *Radiochim. Acta* **99** 405
[17] Anghel C I and Silisteanu I 2017 *Phys. Rev.* C **95** 034611
[18] Ghodsi O N and Hassanzad M 2019 *Nucl. Phys.* A **987** 369
[19] Samanta C, Chowdhary P R and Basu D N 2007 *Nucl. Phys.* A **789** 142
[20] Santhosh K P and Biju R K 2009 *J. Phys. G: Nucl. Part. Phys.* **36** 015107
[21] Poenaru D N, Gherghescu R A and Greiner W 2011 *Phys. Rev.* C **83** 014601
[22] Viola V E and Seaborg G T 1966 *J. Inorg. Nucl. Chem.* **28** 741
[23] Sobiczewski A, Patyk Z and Cwiok S 1989 *Phys. Lett.* B **224** 1
[24] Royer G 2000 *J. Phys. G: Nucl. Part. Phys.* **26** 1149
[25] Royer G 2010 *Nucl. Phys.* A **848** 279
[26] Akrawy D T and Poenaru D N 2017 *J. Phys. G: Nucl. Part. Phys.* **44** 10
[27] Karpov I V *et al* 2012 *Int. J. Mod. Phys.* E **21** 1250013
[28] Brown B A 1992 *Phys. Rev.* C **46** 811
[29] Silisteanu I and Anghel C I 2014 *AIP Conf. Proc.* **1694** 020014
[30] Poenaru D N, Ivascu M, Sandulescu A and Greiner W 1985 *Phys. Rev.* C **32** 572
[31] Poenaru D N and Gherghescu R A 2018 *Phys. Rev.* C **97** 044621
[32] Denisov V Y and Ikezoe H 2005 *Phys. Rev.* C **72** 064613
[33] Myers W D and Swiatecki W J 2000 *Phys. Rev.* C **62** 044610
[34] Dutt I and Puri R K 2010 *Phys. Rev.* C **81** 064609
[35] Ngo H and Ngo C 1980 *Nucl. Phys.* A **348** 140
[36] Qi C, Xu F R, Liotta R J and Wyss R 2009 *Phys. Rev. Lett.* **103** 072501
[37] Silisteanu I and Budaca A I 2012 *At. Data and Nucl. Data Tables* **98** 1096
[38] Budaca A I and Silisteanu I 2013 *Phys. Rev.* C **88** 044618
[39] Xu C, Ren Z and Guo Y 2008 *Phys. Rev.* C **78** 044329
[40] Vandenbosch R and Huizenga J R 1973 *Nuclear Fission* (New York: Academic)
[41] Stoyer M A 2006 *Nature* **442** 876
[42] Muntian I, Hofmann S, Patyk Z and Sobiczewski A 2003 *Acta Phys. Pol. B* **34** 2073 (https://actaphys.uj.edu.pl/R/34/4/2073/pdf)
[43] Muntian I, Patyk Z and Sobiczewski A 2003 *Phys. At. Nucl.* **66** 1015
[44] Le N N *et al* 2020 *J. Radioanal. Nucl. Chem.* **326** 1135
[45] Wang Y Z, Wang S J, Hou Z Y and Gu J Z 2015 *Phys. Rev.* C **92** 064301
[46] Akrawy D T and Ahmed A H 2019 *Phys. Rev.* C **100** 044618
[47] Akrawy D T, Hassanabadi H, Hosseini S S and Santhosh K P 2018 *Nucl. Phys.* A **971** 130
[48] Deng J G, Zhang H F and Royer G 2020 *Phys. Rev.* C **101** 034307
[49] Qian Y and Ren Z 2012 *Phys. Rev.* C **85** 027306
[50] Ni D, Ren Z, Dong T and Xu C 2008 *Phys. Rev.* C **78** 044310
[51] Akrawy D T, Hassanabadi H, Qian Y and Santhosh K P 2019 *Nucl. Phys.* A **983** 310
[52] Koura H 2014 *Prog. Theor. Exp. Phys.* **2014** 113D02
[53] Dong T and Ren Z 2005 *Eur. Phys. J.* A **26** 69
[54] Wang M *et al* 2017 *Chinese Phys.* C **41** 030003
[55] Qi C, Delion D S, Liotta R J and Wyss R 2015 *Phys. Rev.* C **92** 014602
[56] Denisov V Y, Davidovskaya O I and Sedykh I Y 2012 *Phys. Rev.* C **85** 011303
[57] https://t2.lanl.gov/nis/data/astro/molnix96/spidat.html.
[58] Moller P, Sierk A J, Ichikawa T and Sagawa H 2016 *At. Dat. Nucl. Dat. Tables* **109–110** 1
[59] Zagrebaev V I and Greiner W 2015 *Nucl. Phys.* A **944** 257
[60] Baran A *et al* 2015 *Nucl. Phys.* A **944** 442
[61] Perterson D *et al* 2009 *Phys. Rev.* C **79** 044607
[62] Smolanczuk R 1997 *Phys. Rev.* C **56** 812
[63] Smolanczuk R, Skalski J and Sobiczewski A 1995 *Phys. Rev.* C **52** 1871
[64] Swiatecki W J 1955 *Phys. Rev.* **100** 937
[65] Zhang W, Meng J, Zhang S Q, Geng L S and Toki H 2005 *Nucl. Phys.* A **753** 105
[66] Oganessian Y T *et al* 2004 *Phys. Rev.* C **769** 054607
[67] Oganessian Y T *et al* 2006 *Phys. Rev.* C **74** 044602
[68] Oganessian Y T and Utyonkov V K 2015 *Rep. Prog. Phys.* **78** 036301