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A Mathematisation Approach in Teaching Probability

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Abstract

Mathematisation based on "Realistic Mathematics Education" of Freudenthal, which was started in the Netherlands in the 1980s. According to this theory, mathematics is an important and necessary aspect of economic development so education should come from and target to creating skills applied in real-world situations. This paper examines how students solve two real probability situations and suggests measures to integrate the mathematisation process into teaching to develop students' probability literacy. Through experiment, it was found that although students did not know about the mathematisation process, they tended to perform the three steps of this process when facing with a real situation.

Keywords : Mathematisation process, Probability literacy, Teaching probability.

Probability in Vietnam Curriculum

In real life, we often encounter random phenomena, chance circumstances or unpredictable events, relating to probabilistic theory such as finance, weather forecasts, medical risks, bankruptcy, insurance policies, and the ability to win prizes in a game. There has been considerable attention to probability and statistics in school curriculum during the last two decades in many countries for two main reasons (Gal, 2005): (a) probability is one of the important mathematics areas that students are entitled to learn as part of modern education, and (b) probability equips students with the necessary knowledge for life to become probability literate citizens.

In Vietnam, except 1995 series, the former math textbook series presented only combination not probability. Recognizing the important role of probability in modern society, since 2006, probability has been taught to all high school students for "helping students to become familiar with simple probability-related problems in life and science", "step by step bring our high school program to integrate with the world" (Vietnam MOET, 2007). The probability content of Grade 11 Advanced series (Doan, 2006) consists of three parts which are "Event and probability of event", "Probabilistic rules" and "Discrete random variable".

In the probability unit of the textbook, 73.7% probability exercises are set in real-world contexts, however the types of task setting for students to solve are hardly realistic, mainly two types of task 1 and 2 below:

Table 1.
Types of task of probability content in Grade 11 textbook

Order number	Type of task		Mathematics knowledge expected to be used	The number of exercises out of 37 (percentage)
1	Calculate probability		The classical probability definition	18 (47.4%)
			Addition rule and/or multiplication rule of probability.	10 (26.3%)
2	Calculate expectation, variance and standard deviation	Use given probability distribution table.	Formulas of expectation, variance and standard deviation	5 (13.2%)
		Make probability distribution table.		3 (7.9%)
3	Calculate mean value of a discrete random variable X		Formula of expectation.	1 (2.6%)

Looking at Table 1, we can see that most of the tasks focus on calculating skills. Such exercises are necessary but not sufficient for students to select and use appropriate knowledge and skills to solve problems in everyday situations. Furthermore, students will have difficulty dealing with probability situations if two types of task 1 and 2 do not appear explicitly. In addition, it will be hard for students to develop probability literacy if they are only familiar with such types of task. Today, much evidence from research and practice shows that there is no automatic transfer from learning mathematical theory to using that knowledge in situations outside classroom. Students are not able to solve the tasks well in the real world if they do not have the opportunity to confront them (Lovett & Greenhouse, 2000).

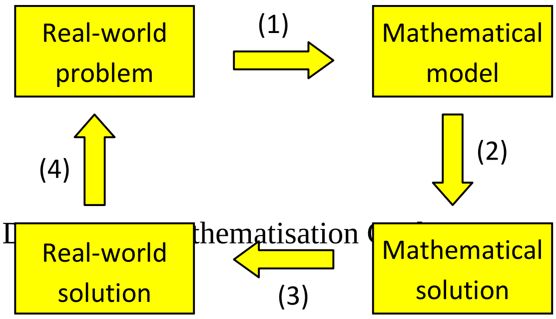
Mathematisation Process

In agreement with Gal’s (2005) viewpoint, "… to develop probability literacy necessitates attention to the issue of skill transfer from in-class learning to situations outside the classroom", we believe that, the mathematisation process is a useful tool to develop students' probability literacy.

Mathematisation process in schools has been increasingly accepted in order to fulfill the objective of strengthening mathematics education towards reality set out by many educationists since the mid-20th century.

Mathematisation process is the process of converting a real problem to a mathematical problem by setting up and solving the mathematical model, and evaluating the solution in real situations, improve model if the solution is not acceptable (PISA, 2006).

Mathematisation is a complex activity, which requires students to have different capacities in different areas of mathematics and contextual knowledge. Many diagrams are used to indicate the nature of the mathematisation activities, as a guide to design the mathematisation tasks and in-class mathematisation implementation (Blum et all, 2007, PISA, 2006, Keiser et all, 2011). A mathematisation cycle relates to the transition between math and reality in both directions and includes four main steps; the steps describe the activities that students will perform.



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Mathematisation process starts with a problem that arises from a real-world situation.

1. Converting the real-world problem to a mathematical problem: determine the necessary mathematical information, recognize mathematical concepts, make the structure, performance estimates related to given situations in mathematical language, describe the nature of elements and relationships in the real-world situations.

2. Working with mathematics: select and use appropriate mathematical methods and tools to solve the problem that has been set up in the mathematical model. Final product at this stage is a mathematical result.

3. Converting the mathematical result to the real-world result: interpret the mathematical result to the original real situation and make sense of the result in the real-world context.

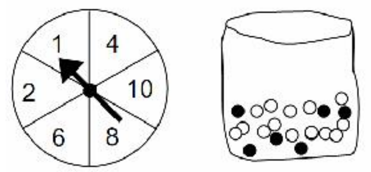
4. Reflecting: review the assumptions and limitations of the model as well as the methods and tools used in solving the problem. This could lead to an improvement in the model as well as the solution or create a new process if necessary.

Method

The study involved two groups of students in grade 11. Each group was composed of 12 students and solved a different situation. We chose two spinner game tasks in this study for two purposes: (a) they include the context from everyday life that is familiar and unsophisticated to students, and (b) when solving the tasks, students need to take a mathematisation process into account.

Situation 1:

A game in a booth at a spring fair involves using a spinner and a marbles bag are represented in the figure below.



Each time, players will turn the spinner and pick up a marble. The player will win if the spinner stop at an even number and he/she gets a black marble from the bag.

Thuy plays the game once. How likely is it that Thuy will win a prize?

A. Impossible.

B. Not very likely (<50%).

C. About 50% likely.

D. Very likely (>50%).

E. Certain.

Situation 2:

In the upcoming camp, class 11A plans to hold a game. Lam suggested using a large spinner wheel with numbers from 1 to 10 on it. To play this game, the player must pay the booth attendant 10,000 VND. The player then chooses two numbers from 1 to 10 and gives the wheel a spin. If the wheel stops on either of the two chosen numbers, the player will get a prize of 60000VND. Do you agree with Lam? Why?

The students, participated in the experiment, had the same educational background – having completed "Combination and probability" unit of Grade 11 advanced, and being totally inexperienced in solving such problems in class. Each group was divided into three small groups and was given 30 minutes to complete the task. When the students expose to the situation, they were required to work individually, and to write down their solution paths, then discussed with each other in group to reach a final solution.

The two researchers were present throughout the experiment, observed groups, made field notes, and asked questions related to mathematisation process for clarification. The interviews were videotaped and student work was collected. In the analysis of study, we focused on steps of the mathematisation process used implicitly by the students when they dealt with a real probability- related situation.

Results

For the situation 1, at first the students were quite embarrassing because the setting of the task is not familiar to the exercises they often see in the textbooks. After reading carefully, all students recognized the important information: the spinner has 6 parts, in which 5 of them are even numbers; the bag has 20 marbles, including 6 black and 14 white; the player win if they come up with an even number and pick out a black marble. At the same time, they also translated the requirement of the situation into "calculating probability of winning in one trial".

To calculate the probability above, students used the following approaches:

- Using the classical probability definition: Probability = number of favorable outcomes / number of possible outcomes. However, many students computed the number of favorable results or the number of possible outcomes wrong.

- The probability to get a prize was the probability that two independent events "stop on an even number" and "and pick a black marble" occurred simultaneously, and students used the multiplication rule of probability. In this step, a few students suggested using addition rule due to confusion in the transition from "and" to "plus operation".

- Using the counting rule: "The ability to come in 5 even numbers was C_6^5, the ability to turn into one of five even numbers was C_5^1, the ability to get 6 black marbles in the bag was C_{20}^6, the ability to get one of six black marbles was C_6^1. So probability $P = C_6^5 \times C_5^1 \times C_{20}^6 \times C_6^1$". This was a mistake in reasoning when solving the problem of two students, but then they realized that this result was not correct, because the value of probability P should belong to the interval [0; 1].

After calculating probability, students answered appropriately to questions posed in real situation, groups chose the answer of "not very likely" if their probability result was less than 0.5, and "very likely" if the result was greater than 0.5, due to mistakes in the solving process. Then they ended their work without any checking. Following is an interview with a student in the group which had the right result.

Teacher: How do you understand this result?

Student: If a player plays the game 4 times, the player will win 1 time.

Teacher: Do you think that there are people who only play one time and win, or play 4 times without winning?

Student: No... but we can win in the first time, this case is too lucky. If we play four times, we certainly win one time because the probability of winning is 0.25 = 1/4.

As you can see in this interview, student's misunderstanding of probability led to his misinterpretation of the result to the initial situation. Although having achieved the right result, this student made a mistake in assuming that the likelihood of a random event was certain with the not large enough number of trials.

When exposing to the situation 2, although it took more time than in scenario 1, students finally understood and grasped the requirement of the situation – they would agree with Lam if his way could make profits. However, when setting up a mathematical model from the given situation, they established the problem of comparing the probability of winning and losing the game. This option did not reflect the nature of the situation because it was not impacted by bet and bonus factors.

When the teacher asked students to compare the average amount paid to the winner with the average amount earned, 4 students thought that they had to compute expectation because of the keyword "average amount" but they were not sure and did not know how to begin.

The teacher suggested: "*X* is the amount paid to the winner. Make a probability distribution table of *X* and calculate expectation E(*X*)". This suggestion was similar to the types of task they encountered in the textbooks, so most students performed well working with mathematics steps. After setting up the table, computing E(*X*), students found that the game would be lose since

$$E(X) = 0.\frac{8}{10} + 60000.\frac{2}{10} = 12000 > 10000 \quad (1)$$

To the question "Which elements of the game should be changed to reverse the situation?", Students based on the formula (1) and made the following comments:

- Increase the amount paid by each player (up 12000 VND);
- Let the player select a number instead of two to reduce the probability of winning to 1/10;
- Reduce the prize for each winner (under 50000 VND);

On so doing, the students learnt the expectation formula but did not understand the meaning of the formula, or not see the link between the concept of expectation with real situations although this is a requirement set in textbook and teacher's edition "E(*X*) is a number gives us an idea of the average magnitude of *X*. So expectation E(*X*) is also known as the average value of *X*". In this example, the expectation E(*X*) = 12000 gave us an imagination, if the game has many players, the average bonus would be 12000 VND per time while the average bet was only 10.000 VND, which means Lam's plan will not make profit for the class.

Discussion and Conclusion

Through this study, we found that although students did not know about the mathematisation process, they tended to perform three steps of this process when facing with a real situation: Conversing real problem to math problem → Working with mathematics → Conversing mathematics result to real result.

(1) Converting a real-world problem to a mathematical problem: students read the relevant factors very carefully to get a clear picture and understand the requirements of the situation, to recognize the valuable information, and the quantities affecting the situation. In each situation, students tried to build the corresponding mathematical models, but sometimes models were not suitable due to the mistakes in students' thinking or reasoning.

(2) Working with mathematics: in two examples, the majority of students did quite well in this step because the situation was stated in mathematics language and the requirements were similar to types of task they had learnt. However there were still mistakes in calculations, using the formula, or counting rules.

(3) Conversing a mathematics result to a real result: after having math results, students only answered questions in real situations without considering the significance of the solution.

Students almost did not have the habit of performing reflection step, but after every real situation, teachers always pose questions to help students think about the reasonableness of solution, and mathematisation process, to find out factors affecting the results, which help students gradually form the reflection habit after each solving process, and also help teachers to have a more completed, comprehensive look at students' understanding.

In conclusion, in teaching probability, teachers do not only teach concepts, formulas, rules, and computing tasks but also bring opportunities for students to be able to understand, explain, and reflect in a real-world probability-related situation, aware the relationship between the probability to reality. To achieve this purpose, the components of probability literacy should be considered and the mathematisation process should be included in teaching implicitly through the examples with the real types of task, in which teachers focus on training students to build mathematical models and reflection step.

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