

Nonlocality function of microscopic optical potentials from Skyrme-based nuclear structure models at low energies

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The nonlocality of the imaginary part of the microscopic optical potential is investigated for neutron elastic scattering on ^{16}O , ^{40}Ca , ^{48}Ca , and ^{208}Pb at incident energies up to 30 MeV. The potential is derived within the particle-vibration coupling framework using fully self-consistent Skyrme-Hartree-Fock plus random-phase approximation (RPA) calculations with the SLy5 interaction, without ad hoc adjustable parameters. By fitting the nonlocality coordinate dependence of the absorptive potential $W(R, s) \equiv \text{Im } \Delta \Sigma$, where $\Delta \Sigma$ is the dynamical part of the nucleon self-energy, to a Gaussian form at representative volume and surface radial positions, we extract the radius- and energy-dependent nonlocality function $\beta(R, E)$, whose values range from 1.01 to 2.03 fm, with a clear dependence on radial position, incident energy, and target mass. These results demonstrate that, for the absorptive part of the optical potential, a single universal constant such as the Perey-Buck value $\beta = 0.85$ fm cannot capture the radial, energy, and target-mass dependence of the nonlocality, and that nucleus-dependent, energy-dependent, and radially resolved descriptions of the absorptive potential W are required for precision nuclear reaction calculations.

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I. INTRODUCTION

In nuclear reaction theory, the nonlocality parameter β plays a central role in the accurate modeling of nucleon-nucleus interactions within the optical potential framework [1–17]. Its origin can be traced to several microscopic mechanisms, including antisymmetrization of wave functions due to the Pauli exclusion principle, finite-range characteristics of the NN interaction, channel couplings, and multiple scatterings. A widely used phenomenological representation of nonlocality is provided by the energy-independent Perey and Buck potential, which employs a Gaussian nonlocality kernel e^{-s^2/β^2} characterized by ad hoc nonlocality parameter $\beta = 0.85$ fm and $s = r - r'$ [8]. This value was determined by fitting elastic neutron scattering data on ^{208}Pb at incident energies of 7 MeV and 14.5 MeV. Despite being constrained by data at only two energies, the Perey-Buck potential offers a remarkably universal framework for describing nucleon-nucleus scattering across a wide range of targets and energies. Once the parameter $\beta = 0.85$ fm was fitted from the ^{208}Pb data, it was applied without further adjustment to calculations on a range of nuclei from Al to Pb and at energies from 0.4 MeV to 24 MeV. This parameter is a key component of the Perey

correction factor, which is employed to simplify calculations, particularly within the distorted wave Born approximation (DWBA), by approximating the nonlocal Schrödinger equation with an equivalent local form. However, the work of Titus *et al.* [4] has shown that the effects of nonlocality cannot be fully captured by the traditional Perey correction factor, highlighting the need for more accurate and systematically derived methods to account for the nonlocal nature of the nucleon-nucleus interaction. This issue becomes particularly critical in the context of modern nuclear reaction models, such as (d, p) and (p, d) reactions in inverse kinematics, which are essential tools for investigating the structure of exotic nuclei. In the unstable nuclei region, the microscopic optical potential is expected to be a useful tool to provide some guidance about nonlocality properties of nucleon-nucleus interaction. Recently, the nonlocal structure of microscopic optical model potentials is calculated in momentum space, which gives a nonlocality form factor featuring a bell shape with nonlocality range between 0.86 and 0.89 fm for both proton and neutron at incident energies below 65 MeV [18,19].

Nonlocality is not merely a phenomenological artifact but a fundamental and unavoidable feature of microscopic optical potentials derived from many-body nuclear theories. Microscopic optical potentials can be derived from a variety of many-body theoretical frameworks that incorporate the complex, nonlocal, and energy-dependent nature of

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nucleon-nucleus interactions. Among these, Green's function theory plays a central role by linking the optical potential to the irreducible nucleon self-energy, naturally embedding nonlocality and dispersion effects. The dispersive optical model (DOM), which builds upon this connection, imposes dispersion relations to constrain the energy dependence of both real and imaginary components of the potential using experimental data [20–24].

Brueckner-Hartree-Fock (BHF) theory constructs optical potentials from in-medium nucleon-nucleon G -matrix interactions folded over nuclear densities, providing insight into medium modifications and density dependencies [18,25]. Additionally, *ab initio* approaches including the self-consistent Green's function (SCGF) and coupled-cluster methods yield optical potentials consistent with the same underlying nuclear forces used in structure calculations [26–28]. These frameworks collectively demonstrate that nonlocality is a robust and intrinsic feature of the nucleon self-energy and must be incorporated for a consistent description of nuclear reactions.

At low incident energies, where specific nuclear structure effects, such as collective vibrational excitations, play a significant role, Vinh Mau derived the optical potential $U(\mathbf{r}, \mathbf{r}'; E)$ from the mass operator or self-energy $\Sigma(\mathbf{r}, \mathbf{r}'; E)$ including random-phase approximation (RPA) states, leading to an inherently nonlocal, energy-dependent, and complex potential that reflects the underlying nucleon correlations and exchange mechanisms within the nucleus [29]. In this framework, a simple zero-range effective nucleon-nucleon interaction was employed to model the imaginary part of the potential for neutron scattering on ^{40}Ca at low energies [30]. The nonlocal nature of the resulting microscopic optical potential (MOP) was investigated in both the surface and volume regions of the nucleus.

Later, this approach was extended to the finite-range Gogny effective interactions, which ensure a realistic representation of the nucleon-nucleon interaction, capturing the essential features that contribute to nonlocality [31–33]. By inherently incorporating nonlocality and energy dependence, this approach offers an accurate and physically grounded description of nucleon-nucleus interactions. A Perey-Buck equivalent potential has been fitted by using the microscopic optical potential generated from effective Gogny interaction for ^{40}Ca and ^{48}Ca . These calculations can give valuable guidance for further nonlocal potential parametrizations. However, due to the heavy numerical calculation using Gogny interaction, the modest calculations are limited to the region of medium nuclei [33].

Concurrently with the calculations using the Gogny interaction, similar studies were conducted by seminal work by Bernard *et al.* [34], employing Skyrme effective zero-range interactions. The nonlocality parameter β is also extracted by the Gaussian shape of the absorption part of the microscopic optical potential: $W(R, s) = W(R, s=0) \times e^{-s^2/\beta^2}$, where $R = \frac{r+r'}{2}$. By performing the calculations for neutron elastic scattering off ^{208}Pb at energies below 30 MeV, they found β_s varies slightly from 0.98 fm at $E = 5$ MeV to 0.92 fm at $E = 30$ MeV only at the peak radius $R = 7.2$ fm on the surface. In the nuclear interior, it is not possible to extract the nonlocality parameter β_v due to the absence of absorptive strength.

Based on the work of Ref. [34], recent advances in computational power and many-body theoretical methods [35–37] allow us to use the effective Skyrme interaction consistently at all levels, from the mean-field description to the treatment of correlations beyond the mean field in a model space which includes all natural parity RPA excited states with multipolarity $L = 0^+, 1^-, 2^+, 3^-, 4^+, 5^-$ and excitation energies below 50 MeV. In conjunction with approaches employing the Gogny interaction, these developments offer a unified and self-consistent framework for describing nuclear structure and reactions at low energies. In comparison with the results of Ref. [34], the calculated absorption is not only enhanced at the nuclear surface but also exhibits an important contribution in the nuclear interior for all doubly closed-shell nuclei ^{16}O , ^{40}Ca , ^{48}Ca , and ^{208}Pb at incident energies below 50 MeV [38–40]. Building on this progress, as a first step, we focus on investigating the nonlocality parameter β from the nuclear interior to the surface for a range of target nuclei, including ^{16}O , ^{40}Ca , ^{48}Ca , and ^{208}Pb , at incident energies below 50 MeV.

II. THEORETICAL FRAMEWORK

According to Refs. [30,34,38–40], we define the quantity $W(\mathbf{R}, \mathbf{s}, E) \equiv \text{Im}\Delta\Sigma(\mathbf{r}, \mathbf{r}', E)$ as a function of $\mathbf{R} = \frac{1}{2}(\mathbf{r} + \mathbf{r}')$, which corresponds to the radius and shape of $\text{Im}\Delta\Sigma$, and $\mathbf{s} = \mathbf{r} - \mathbf{r}'$, which characterizes its nonlocality. The $\Delta\Sigma(\mathbf{r}, \mathbf{r}', E)$ is nonlocal, complex, and energy dependent giving a major contribution to the imaginary part which is responsible for the absorption of the optical potential

$$V_{\text{opt}}(\mathbf{r}, \mathbf{r}', E) = V_{\text{HF}}(\mathbf{r}) + \Delta\Sigma(\mathbf{r}, \mathbf{r}', E), \quad (1)$$

where

$$\Delta\Sigma(\mathbf{r}, \mathbf{r}', E) = \Sigma(\mathbf{r}, \mathbf{r}', E) - \frac{1}{2}\Sigma^{(2)}(\mathbf{r}, \mathbf{r}', E). \quad (2)$$

In the general case, W is a very complicated function of r , r' and θ the angle between the two vectors $\mathbf{r}$ and $\mathbf{r}'$.

The potential can be expanded as

$$W(\mathbf{R}, \mathbf{s}, E) = \sum_{\Lambda=0}^{\infty} W_{\Lambda}(s, R, E) P_{\Lambda}(\cos\theta), \quad (3)$$

where P_{Λ} are Legendre polynomials. Vinh Mau and Bouyssy [30] demonstrated that the term $\Lambda = 2$ of the potential, and *a fortiori* terms with $\Lambda > 2$, is much smaller than the first term $\Lambda = 0$, at least by an order of magnitude. Therefore, to a very good approximation one can write $W(\mathbf{R}, \mathbf{s}, E) = W_0(R, s, E)$. This independence of W upon θ justifies treating $R = |\mathbf{R}|$ and $s = |\mathbf{s}|$ as scalar quantities (magnitudes) in our analysis. This is the standard convention universally adopted in phenomenological nonlocal optical potentials, including the seminal Perey-Buck potential [8] as well as the Engelbrecht-Fiedeldey parametrization [41]. In both cases, s and R are explicitly treated as scalar variables representing the magnitudes of the relative and center-of-mass coordinates, respectively.

In Eq. (1), V_{HF} is a static Skyrme-Hartree-Fock mean field which is real, local, and energy independent, providing a major contribution to the real part of the optical potential. To take into account the issue of the Pauli principle correction,

$\Sigma^{(2)}(E)$ is the second-order potential (SOP) generated from uncorrelated particle-hole contribution, and E is the nucleon incident energy. When a zero-range interaction such as the Skyrme force is employed, the nonlocality of the optical potential originates exclusively from the dynamical component of the self-energy, $\Delta\Sigma(E)$. This contrasts with the case of finite-range interactions, such as the Gogny force, where nonlocal features are present in both the static mean-field contribution, V_{HF} , and the polarization term $\Delta\Sigma(E)$. However, the use of zero-range Skyrme interactions significantly decreases the numerical complexity of the calculations, which nevertheless remain feasible for nuclei ranging from light to heavy targets. The effective SLy5 Skyrme interaction [42] has been chosen.

Following the convention adopted in Ref. [34], the dependence of the absorptive potential $W(R, s, E)$ on the nonlocality coordinate s , at a fixed radial position R and incident energy E , is represented by the Gaussian form

$$W(R, s, E) = W(R, s = 0, E) \exp\left[-\frac{s^2}{\beta^2(R, E)}\right], \quad (4)$$

where the width $\beta(R, E)$ characterizes the curvature of $W(R, s)$ as a function of s at fixed R and E . Hence, β is not a single universal constant but a radius- and energy-dependent function, $\beta \equiv \beta(R, E)$. Throughout this work we keep the conventional name “nonlocality parameter”, in line with the Perey-Buck terminology [8], to denote the value of this function at a given (R, E) ; these values are reported at the representative volume (R_v) and surface (R_s) radii for incident energies between 5 and 30 MeV. We also note that the present analysis is restricted to the imaginary part of the dynamical self-energy, $W \equiv \text{Im } \Delta\Sigma$, which governs the absorption of flux from the elastic channel; the nonlocality of the real dynamical component $\text{Re } \Delta\Sigma$ is not addressed here, and all statements on the nonlocality made in this work therefore refer to the absorptive part only.

For the description of doubly magic nuclei, spherical symmetry is adopted to simplify the numerical calculations. First, we solved the radial Hartree-Fock (HF) equations in coordinate space by using a mesh size of 0.1 fm with the radial coordinate extending up to 15 fm. Ground and excited states are constructed on top of the HF solution using fully self-consistent RPA calculations [36]. All natural-parity RPA excited states with multipolarities $L = 0^+, 1^-, 2^+, 3^-, 4^+, 5^-$ and excitation energies below 50 MeV are selected for the particle-vibration coupling (PVC) calculations [35,36]. Continuum states with positive energy are treated by discretizing the spectrum using box boundary conditions. All computational parameters are kept fixed for all nuclei and excitation energies considered, following the methodology of Refs. [38–40]. It is worth noting that several numerical codes are now available for solving the integrodifferential Schrödinger equation with nonlocal optical potentials, as reviewed by Hebborn *et al.* [43]. Among these, the code SIDES [44] provides a dedicated solver for nonlocal potentials in coordinate space. In our previous works [38–40], the nonlocality of the microscopic optical potential is treated exactly by employing the distorted wave Born approximation (DWBA)

code developed by Raynal [45], which solves the full integrodifferential equation for the scattering wave function in the presence of a nonlocal potential.

To keep the present study self-contained and to illustrate the quality of the underlying microscopic potential, Fig. 6 (adapted from Ref. [38]) shows the neutron elastic-scattering differential cross section for ^{208}Pb at $E = 14.5$ MeV, obtained from the same fully self-consistent particle-vibration-coupling optical potential whose nonlocality is characterized in this work. The scattering observables are computed by solving the nonlocal Schrödinger equation exactly with the DWBA98 code [45] without introducing any local-equivalent (Perey-type) approximation. The figure displays the cumulative contributions of the collective multipole modes, obtained by progressively increasing the maximum multipolarity L_{max} from 1 to 5, together with the static HF result and the experimental data. The dominant contributions to the absorption originate from the most collective 3^- , 4^+ , and 2^+ states, whereas the 1^- and 5^- modes play a minor role; once all natural-parity modes are included, the measured angular distribution is well reproduced. A systematic account of the angular distributions for all four target nuclei ^{16}O , ^{40}Ca , ^{48}Ca , and ^{208}Pb over the energy range considered here was reported in our previous works [38–40].

III. RESULTS AND DISCUSSION

We begin by calculating the imaginary part of the nonlocal optical potential, $W(R, s)$, as a function of the radial coordinate R , for fixed nonlocality coordinate $s = 0, 0.2, 0.4, 0.6, 0.8, 1.0$ fm. These calculations are performed for the ^{16}O , ^{40}Ca , ^{48}Ca , and ^{208}Pb nuclei at various incident energies. As illustrated in Figs. 1, 2, 3, 4, the imaginary part $W(R, s)$, which characterizes the absorptive strength of the potential, systematically decreases with increasing nonlocality coordinate s . For the light nucleus ^{16}O and the medium-mass nuclei ^{40}Ca and ^{48}Ca , the imaginary part of the optical potential exhibits distinct volume and surface components at all investigated incident energies, with the exception of ^{16}O at 13.5 and 22 MeV. For the heavy nucleus ^{208}Pb , the surface potential is dominant, except at high energies $E = 30$ MeV for $s = 0.8, 1.0$ fm where a minimum appears in the volume region. As a function of s , $W(R, s)$ is close to a Gaussian for all incident energies and radii considered.

We note that the Gaussian approximation holds reasonably well at the nuclear surface for the low-energy regime considered in this work, consistent with the findings of Blanchon *et al.* [32], who showed good agreement between the nonlocality of their microscopic NSM/Gogny potential and a Gaussian shape with $\beta \approx 0.94$ fm at $r = 4.3$ fm for neutron scattering from ^{40}Ca at 9.91 MeV. However, we acknowledge that a purely Gaussian nonlocality does not capture the full complexity revealed by microscopic studies. In particular, Arellano and Blanchon [19] disclosed a universal separable structure of the optical potential in which the nonlocality form factor is inherently complex and of bell-shape (hydrogenic) type, decaying as $e^{-s/b}$ rather than the Gaussian e^{-s^2/β^2} . This bell-shape nonlocality was found to be a robust feature across targets (^{40}Ca , ^{90}Zr , ^{208}Pb) and beam energies (40–400 MeV),

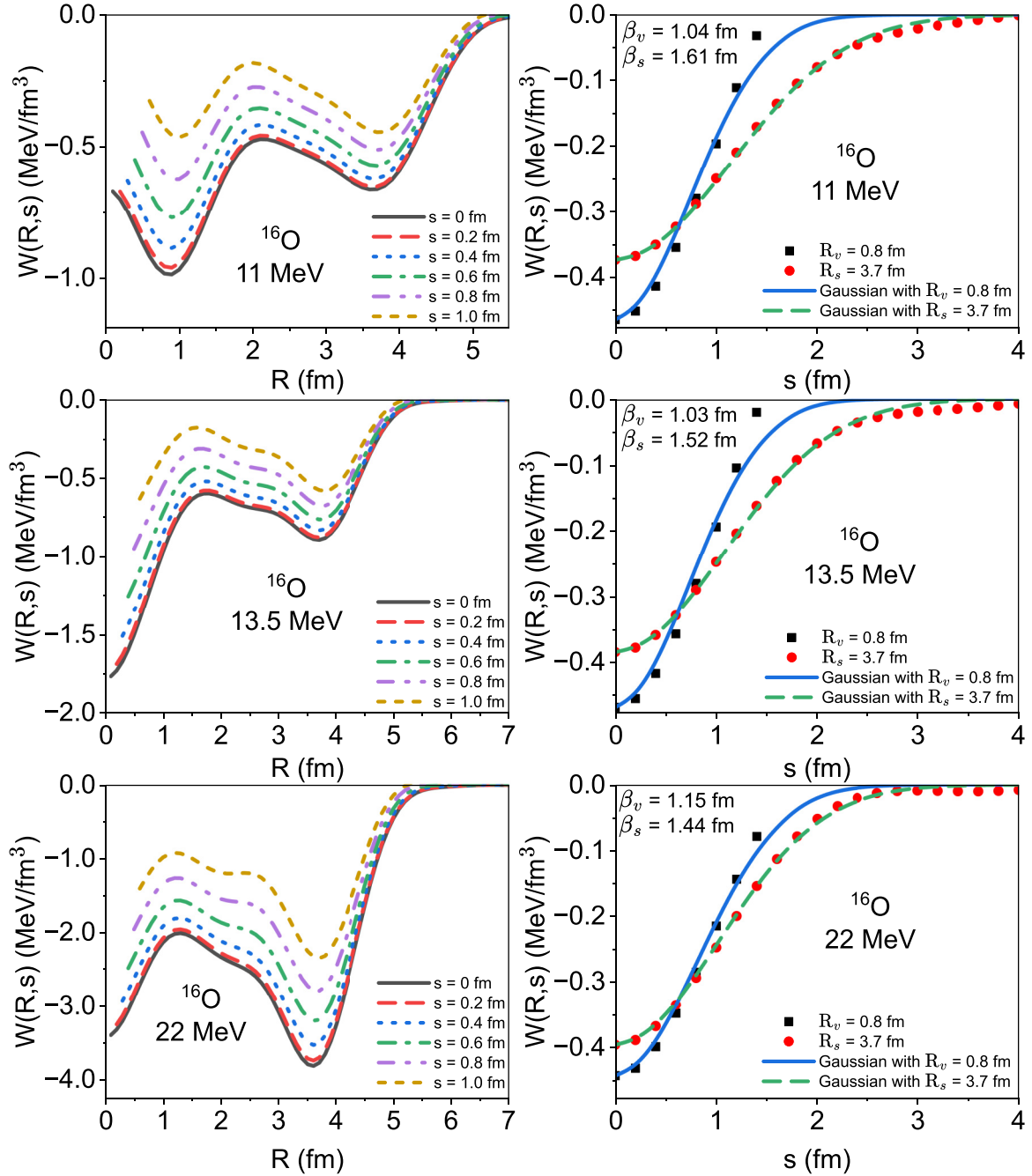


FIG. 1. Imaginary part of the nonlocal optical potential $W(R, s)$ and extracted nonlocality parameter β for neutron elastic scattering on ^{16}O at incident energies of 11 MeV, 13.5 MeV, and 22 MeV. Left panels: $W(R, s)$ as a function of the center-of-mass coordinate R for various fixed nonlocality coordinates s . Right panels: $W(R, s)$ as a function of the nonlocality coordinate s at representative radial positions R_v (black squares, volume region) and R_s (red circles, surface region). The blue solid curve represents the Gaussian fit for the volume region with the extracted β_v , while the green dashed curve represents the Gaussian fit for the surface region with the extracted β_s .

exhibiting a slower fall off and longer tail compared to the Gaussian form. Furthermore, Arellano and Blanchon [46] demonstrated that the nonlocal structure in coordinate space is not unique: different momentum cutoffs applied to the same momentum-space potential yield different-looking nonlocalities while preserving identical scattering observables, implying that shape comparisons must be interpreted with caution. The approximate Gaussian behavior found in our

Skyrme-based calculations at low energies ($E < 30$ MeV) is therefore understood as a valid surface-dominated description within the energy range studied, while a more complete characterization of the nonlocality, particularly in the nuclear interior and at higher energies, would require adopting the bell-shape form factor or similar non-Gaussian prescriptions.

To quantify the quality of the Gaussian fit of $W(R, s)$ from which β is extracted, we define the coefficient of

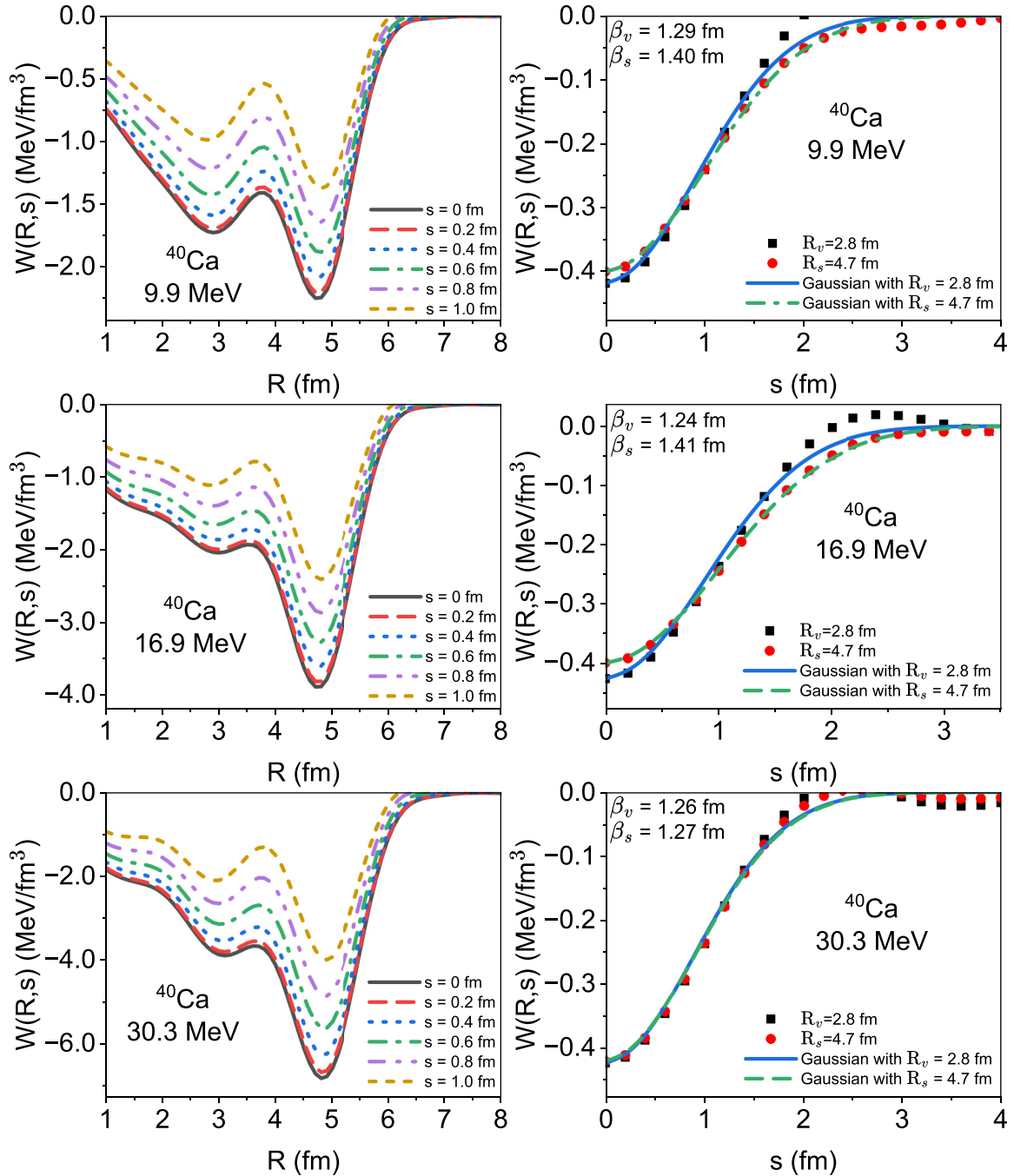


FIG. 2. Same as Fig. 1, but for ^{40}Ca at incident energies of 9.9 MeV, 16.9 MeV, and 30.3 MeV.

determination, denoted $\mathcal{R}^2$, which measures how well the Gaussian model explains the calculated microscopic data. The $\mathcal{R}^2$ value is computed using

$$\mathcal{R}^2 = 1 - \frac{\text{SS}_{\text{res}}}{\text{SS}_{\text{tot}}}, \quad (5)$$

where

$$\text{SS}_{\text{res}} = \sum_{i=1}^N [W_{\text{calc}}(R, s_i) - W_{\text{fit}}(R, s_i)]^2 \quad (6)$$

is the residual sum of squares, and

$$\text{SS}_{\text{tot}} = \sum_{i=1}^N [W_{\text{calc}}(R, s_i) - \bar{W}]^2 \quad (7)$$

is the total sum of squares. Here, $W_{\text{calc}}(R, s_i)$ denotes the calculated W at the fixed radial position R and nonlocality coordinate s_i , $W_{\text{fit}}(R, s_i)$ is the corresponding fitted Gaussian value, $\bar{W} = N^{-1} \sum_{i=1}^N W_{\text{calc}}(R, s_i)$ is the mean of the calculated data at that radius, and N is the number of data points. Table I summarizes the extracted nonlocality param-

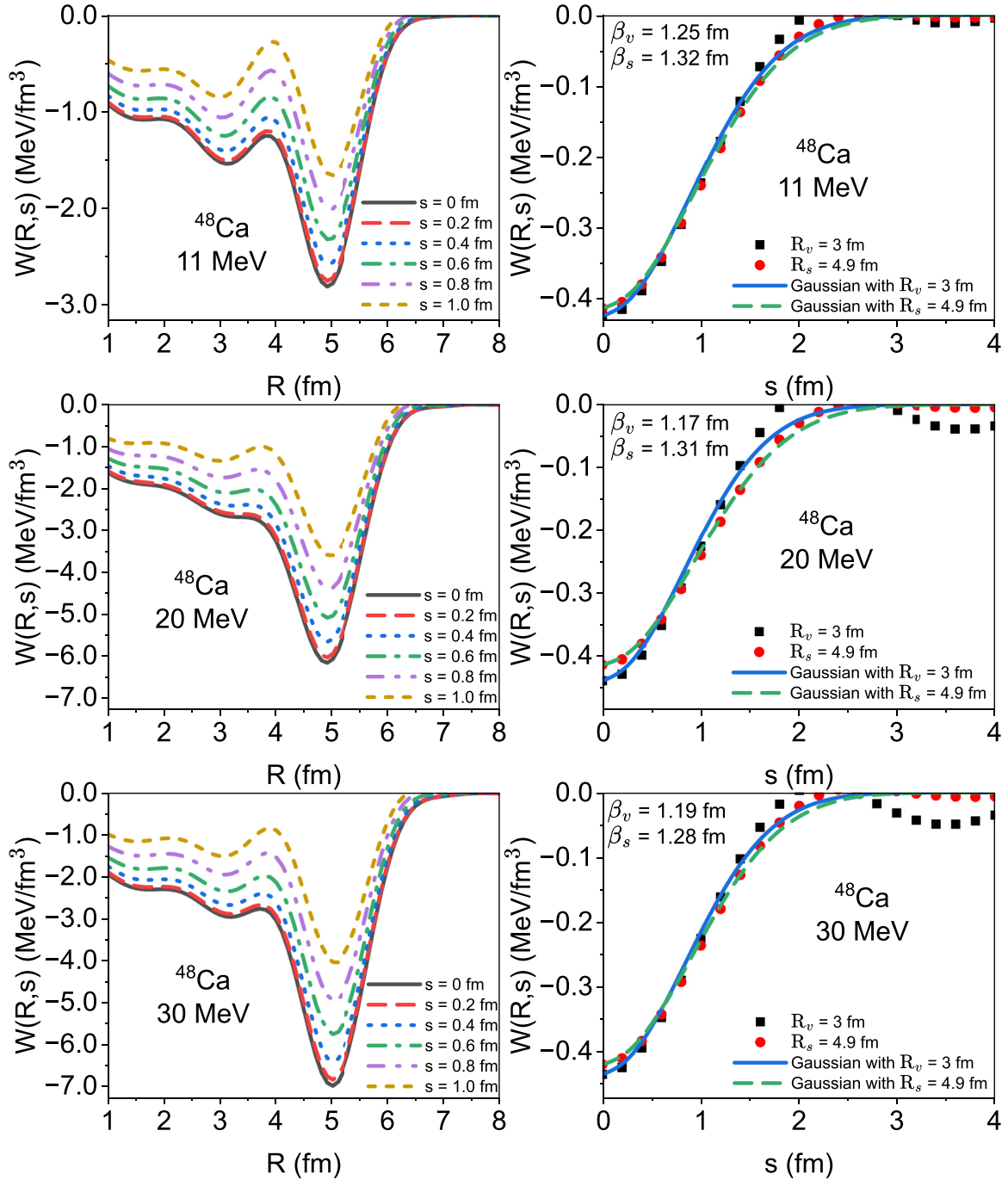


FIG. 3. Same as Fig. 1, but for ^{48}Ca at incident energies of 11 MeV, 20 MeV, and 30 MeV.

ters β along with the corresponding $\mathcal{R}^2$ values for all target nuclei and incident energies presented in Figs. 1–4. The obtained values of $\mathcal{R}^2$ range from 0.970 to 0.999, confirming that the Gaussian ansatz of Eq. (4) provides a very good representation of W across all systems and energies considered. The fits are systematically better in the surface region ($\mathcal{R}^2 \geq 0.99$) than in the nuclear interior, where minor deviations from the Gaussian form appear at large values of the nonlocality coordinate s . Our calculations with zero-range Skyrme interactions confirm that the Gaussian shape of the

imaginary-potential nonlocality is independent of the form factors of the underlying interaction. Figure 5 presents the energy dependence of the nonlocality parameter β for neutron elastic scattering on ^{16}O , ^{40}Ca , ^{48}Ca , and ^{208}Pb , evaluated at representative radial positions corresponding to the nuclear interior (β_v) and surface (β_s) regions, over the incident energy range $E = 5$ –30 MeV.

For the light nucleus ^{16}O , a pronounced separation between surface and volume nonlocality is observed across the entire energy range. The surface parameter β_s at $R = 3.7$ fm decreases from approximately 1.75 fm at $E \approx 6.5$ MeV to about

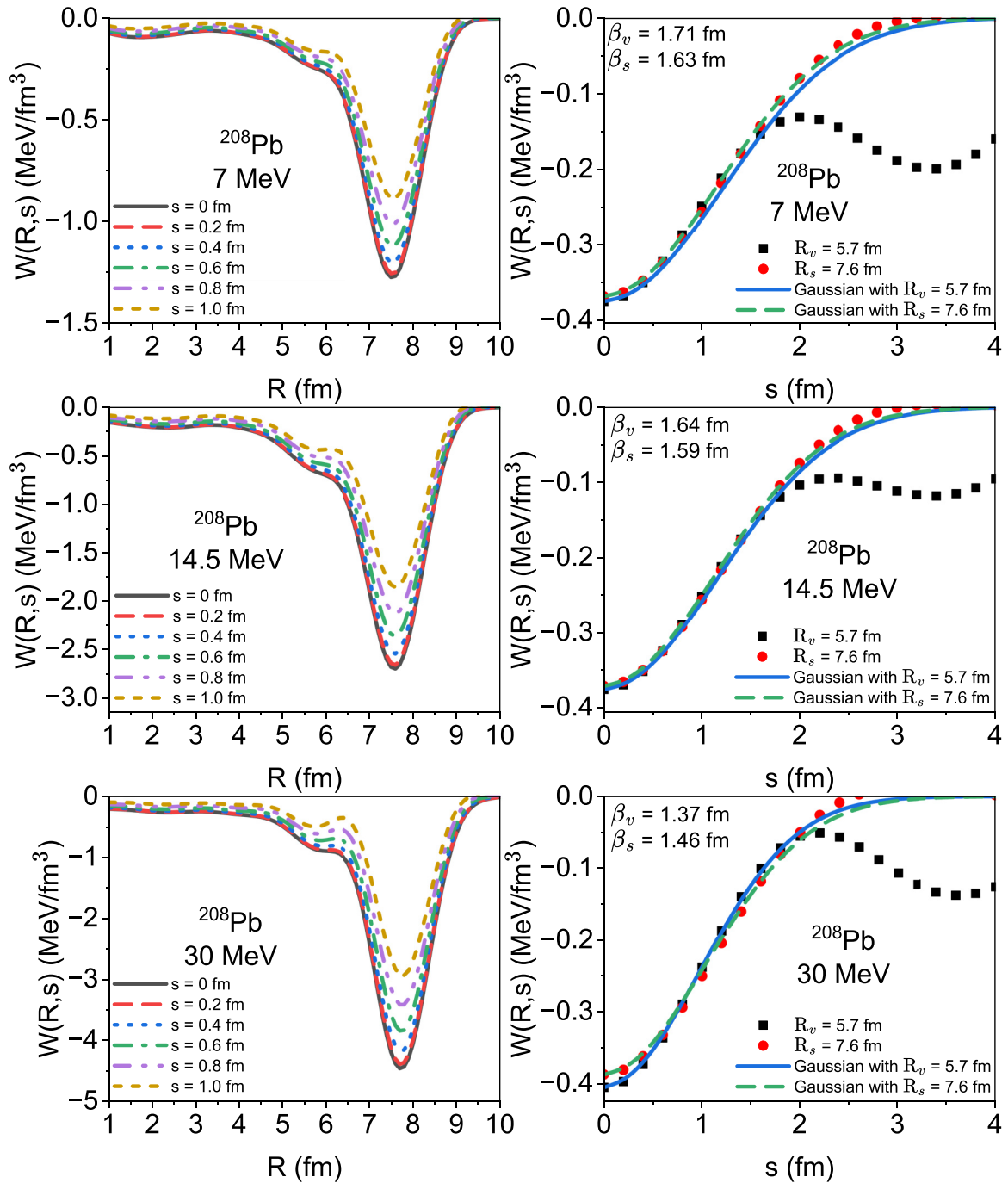


FIG. 4. Same as Fig. 1, but for ^{208}Pb at incident energies of 7 MeV, 14.5 MeV, and 30 MeV.

1.36 fm at $E \approx 30$ MeV, exhibiting a monotonic decreasing trend with a total variation of roughly 22.4%. In contrast, the volume parameter β_v at $R = 0.8$ fm remains comparatively stable around 1.01–1.20 fm, with weak fluctuations and no pronounced energy trend. The gap $\Delta\beta = \beta_s - \beta_v$ ranges from approximately 0.24 to 0.72 fm with a typical value of about 0.37 fm, indicating that the nonlocal structure in ^{16}O is governed by qualitatively different mechanisms in the two regions: the interior is dominated by mean-field correlations and Pauli blocking, which constrain the nonlocality range, while the surface is more sensitive to particle-vibration coupling

with low-lying collective states, leading to a substantially broader nonlocal extent.

For ^{40}Ca , both β_v ($R = 2.8$ fm) and β_s ($R = 4.7$ fm) exhibit an initial peak near $E \approx 6$ –7 MeV, reaching ≈ 1.43 fm and ≈ 1.56 fm, respectively, followed by a gradual decrease toward higher energies. Above $E \approx 20$ MeV, both parameters stabilize around 1.25–1.34 fm. The difference between surface and volume values is significantly reduced compared to ^{16}O , with $\Delta\beta \lesssim 0.24$ fm over the full energy range considered, converging to $\Delta\beta \approx 0.01$ fm at $E = 30.3$ MeV, where $\beta_v = 1.26$ fm and $\beta_s = 1.27$ fm. This near-degeneracy at higher energies

TABLE I. Extracted nonlocality parameters β (in fm) and fit quality $\mathcal{R}^2$ for neutron elastic scattering on ^{16}O , ^{40}Ca , ^{48}Ca , and ^{208}Pb at various incident energies.

Nucleus	E (MeV)	β_v (fm)	$\mathcal{R}_v^2$	β_s (fm)	$\mathcal{R}_s^2$
^{16}O	11.0	1.04	0.981	1.61	0.999
	13.5	1.03	0.976	1.52	0.998
	22.0	1.15	0.993	1.44	0.999
^{40}Ca	9.9	1.29	0.992	1.40	0.998
	16.9	1.24	0.994	1.41	0.999
	30.3	1.26	0.992	1.27	0.997
^{48}Ca	11.0	1.25	0.995	1.32	0.998
	20.0	1.17	0.979	1.31	0.998
	30.0	1.19	0.970	1.28	0.997
^{208}Pb	7.0	1.71	0.971	1.63	0.999
	14.5	1.64	0.994	1.59	0.998
	30.0	1.37	0.999	1.46	0.996

reflects the more uniform nuclear density profile and stronger shell closure in ^{40}Ca , which produces a more homogeneous nonlocal environment throughout the nuclear volume.

It is instructive to compare these results with the pioneering microscopic calculation of Vinh Mau and Bouyssy [30], who investigated the nonlocality of the imaginary optical potential for neutron- ^{40}Ca scattering using a zero-range effective nucleon-nucleon interaction within the Green

function formalism including RPA correlations. In their work, the nonlocality range was evaluated at $R = 1$ fm for the nuclear interior and $R = 3.5$ fm for the surface, yielding $\beta_v \approx 1$ fm (nearly energy-independent) and β_s decreasing from 1.58 fm to 1.36 fm as E increases from 8 to 25 MeV.

In the present calculation, the volume nonlocality β_v is evaluated at $R = 2.8$ fm and ranges from 1.24 to 1.29 fm at the energies shown in Table I, reaching ≈ 1.43 fm only at the lowest energies ($E \approx 6$ MeV) as displayed in Fig. 5. The somewhat larger β_v compared to the value of ≈ 1 fm reported in Ref. [30] may be attributed to the different radial positions at which the nonlocality is probed, our evaluation at $R = 2.8$ fm lies closer to the surface region of ^{40}Ca than the deep interior point $R = 1$ fm used in Ref. [30], where the density is higher and more uniform and to the fully self-consistent treatment and the substantially larger RPA model space employed here. Nevertheless, the nearly energy-independent character of β_v reported in Ref. [30] is qualitatively confirmed by our results, which show only a modest $\approx 3.3\%$ variation (from 1.29 to 1.24 fm) over the energy range $E = 9.9$ –30.3 MeV.

The most notable difference concerns the surface nonlocality β_s . Our values, ranging from 1.27 to 1.41 fm, are of the same order as, although still somewhat smaller than, the 1.36–1.58 fm reported in Ref. [30]. Moreover, the energy dependence of β_s in our calculations is considerably weaker: the total variation is about 9.1% (from 1.40 to 1.27 fm over $E = 9.9$ –30.3 MeV), compared to the $\approx 14\%$ decrease (from

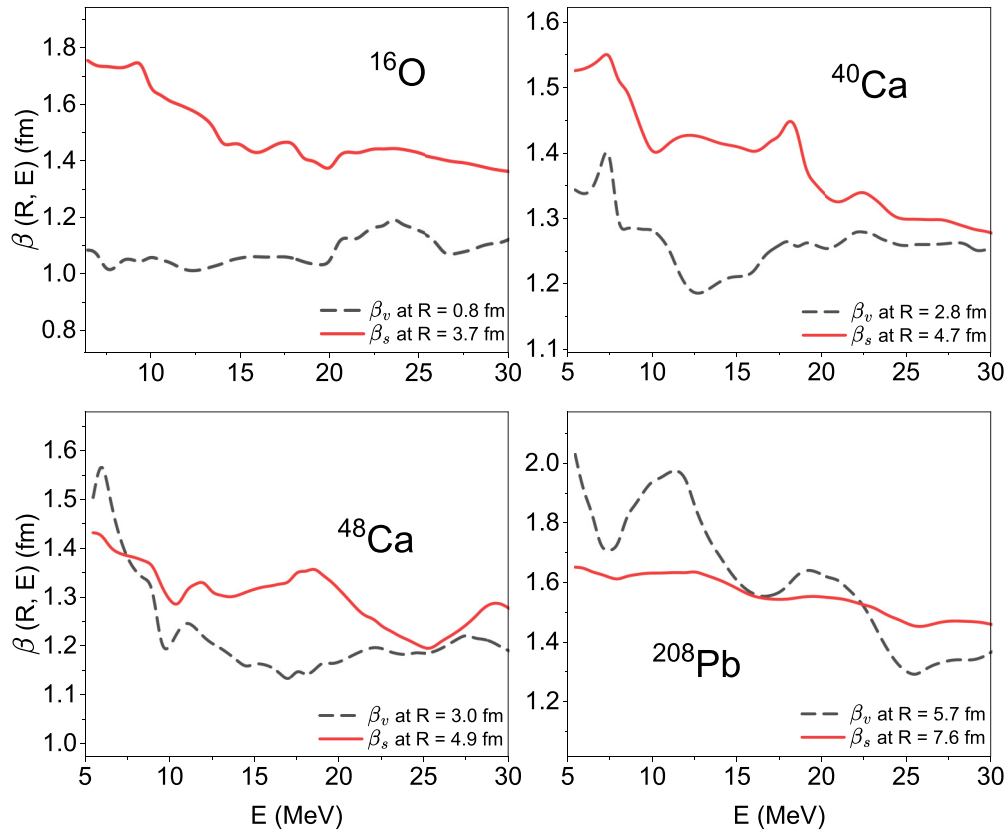


FIG. 5. Energy dependence of the nonlocality parameter β for neutron elastic scattering on ^{16}O , ^{40}Ca , ^{48}Ca , and ^{208}Pb over the incident energy range $E = 5$ –30 MeV. The gray dashed line represents the volume nonlocality parameter β_v , while the red solid line represents the surface nonlocality parameter β_s .

1.58 to 1.36 fm over $E = 8\text{--}25\text{ MeV}$ found in Ref. [30]. Several factors may account for the remaining difference. First, the surface evaluation radii differ: our $R_s = 4.7\text{ fm}$ lies further from the nuclear center than the $R = 3.5\text{ fm}$ used in Ref. [30], which was constrained by the smaller harmonic oscillator parameter ($b_0 = 1.89\text{ fm}$, corresponding to a nuclear radius of only 3.2 fm) employed in their calculation. Second, and more importantly, the present framework employs fully self-consistent Skyrme-Hartree-Fock plus RPA calculations with the SLy5 interaction, incorporating all natural-parity states with multipolarities $L = 0^+, 1^-, 2^+, 3^-, 4^+, 5^-$ and excitation energies up to 50 MeV . In contrast, Ref. [30] used a phenomenological zero-range force with Soper exchange mixture and a more restricted set of RPA states, without self-consistency between the mean field and the residual interaction. The residual difference in the surface nonlocal range may therefore reflect the limited model space and the particular choice of interaction parameters of Ref. [30] rather than a genuine physical effect; the overall magnitude of the surface nonlocality obtained in the two calculations is, however, in reasonable agreement. Despite these quantitative differences, both calculations share the fundamental qualitative result that the imaginary part of the nonlocal potential exhibits a Gaussian dependence on the nonlocality coordinate s , confirming that this functional form is independent of the specific form factors of the underlying zero-range interaction.

The neutron-rich isotope ^{48}Ca reveals a more complex behavior. At very low energies ($E < 7\text{ MeV}$), the volume parameter β_v at $R = 3.0\text{ fm}$ exceeds the surface parameter β_s at $R = 4.9\text{ fm}$, with β_v reaching approximately 1.60 fm . With increasing energy, β_v decreases rapidly to $\approx 1.13\text{--}1.25\text{ fm}$, while β_s stabilizes around $1.27\text{--}1.32\text{ fm}$, leading to a crossover at $E \approx 7\text{--}8\text{ MeV}$ above which β_s consistently exceeds β_v . Furthermore, β_s exhibits a nonmonotonic structure with a local maximum around $E \approx 18\text{--}20\text{ MeV}$ reaching $\approx 1.31\text{--}1.36\text{ fm}$ before decreasing at higher energies. This nontrivial energy dependence may be attributed to the neutron excess in ^{48}Ca , which modifies the spectrum of collective excitations and introduces additional isovector modes that couple differently to the scattering nucleon in the interior and at the surface. The crossing of β_v and β_s as a function of energy is a distinctive feature of this neutron-rich system. As shown in Fig. 3, the difference $\Delta\beta = \beta_s - \beta_v$ remains modest, ranging from $\approx 0.07\text{ fm}$ at $E = 11\text{ MeV}$ to $\approx 0.15\text{ fm}$ at $E = 20\text{ MeV}$ and $\approx 0.09\text{ fm}$ at $E = 30\text{ MeV}$.

For the heavy nucleus ^{208}Pb , the most striking features among all investigated targets are observed. As shown in Fig. 5, the volume parameter β_v at $R = 5.7\text{ fm}$ exhibits a strong energy dependence, starting at approximately 2.03 fm at $E \approx 5\text{ MeV}$, displaying pronounced fluctuations with peaks reaching $\approx 1.98\text{ fm}$ near $E \approx 10\text{--}15\text{ MeV}$, and then decreasing steeply to $\approx 1.37\text{ fm}$ at $E = 30\text{ MeV}$. In contrast, the surface parameter β_s at $R = 7.6\text{ fm}$ shows a nonmonotonic behavior: it starts at $\approx 1.65\text{ fm}$ at $E \approx 5\text{ MeV}$, exhibits a dip around $E \approx 8\text{--}9\text{ MeV}$, recovers to $\approx 1.54\text{--}1.58\text{ fm}$ at intermediate energies ($E \approx 15\text{--}20\text{ MeV}$), and then settles around $\approx 1.46\text{--}1.47\text{ fm}$ at $E = 30\text{ MeV}$. Despite these fluctuations, β_s remains considerably more stable than β_v over the full energy range. Remarkably, at low energies β_v significantly

exceeds β_s . The crossing between β_v and β_s occurs around $E \approx 22\text{--}22.5\text{ MeV}$, above which the conventional hierarchy $\beta_s > \beta_v$ is restored.

We next compare in detail with the seminal work of Bernard and Van Giai [34] on nucleon- ^{208}Pb scattering. In Ref. [34], the imaginary part of the optical potential was calculated using the Skyrme SIII interaction within a framework where the same effective force was used to generate the Hartree-Fock field, the RPA excited states, and the particle-vibration coupling. The nonlocality of $W(R, s)$ was analyzed at the peak radius $R = 7.2\text{ fm}$, where a Gaussian shape was found with β_s varying slightly from 0.98 fm at $E = 5\text{ MeV}$ to 0.92 fm at $E = 30\text{ MeV}$. Crucially, in Ref. [34] the absorptive potential was found to vanish in the nuclear interior, making it impossible to extract a volume nonlocality parameter β_v . This absence of interior absorption was attributed to two reinforcing effects: the strong density dependence of the SIII coupling strength $\bar{v}(r)$, which becomes very small inside the nucleus, and the surface-peaked nature of the transition densities $\rho_N(r)$ for the dominant low-lying isoscalar collective states. Furthermore, the calculation of Ref. [34] excluded spin-orbit, velocity-dependent (t_1, t_2), and spin-dependent terms in the residual interaction, and was restricted to a limited set of vibrational modes (isoscalar states with $L = 0^+, 2^+, 3^-, 4^+, 5^-$ and isovector states with $L = 0^+, 1^-, 2^+$), which may have further limited the completeness of the coupling scheme in the nuclear interior.

In contrast, the present calculation based on the SLy5 Skyrme interaction with fully self-consistent Hartree-Fock plus RPA, including all natural-parity states with $L = 0^+, 1^-, 2^+, 3^-, 4^+, 5^-$ and excitation energies up to 50 MeV , reveals significant absorptive strength not only at the surface but also in the nuclear interior for all incident energies considered. This is evident from the left panels of Fig. 4, where $W(R, s = 0)$ exhibits clear volume contributions centered around $R \approx 5\text{--}6\text{ fm}$, particularly at higher energies. The presence of interior absorption now enables, for the first time in a Skyrme-based PVC framework applied to ^{208}Pb , the extraction of the volume nonlocality parameter β_v .

Regarding the surface nonlocality, our extracted values $\beta_s = 1.63\text{ fm}$ at $E = 7\text{ MeV}$, 1.59 fm at $E = 14.5\text{ MeV}$, and 1.46 fm at $E = 30\text{ MeV}$ (Table I), evaluated at $R_s = 7.6\text{ fm}$, are systematically larger than the values of $0.98\text{--}0.92\text{ fm}$ reported in Ref. [34] at $R = 7.2\text{ fm}$.

Nevertheless, both calculations exhibit the same qualitative trend of a decreasing β_s with increasing incident energy. The quantitative difference can be traced to several factors: the use of SLy5 versus SIII, the substantially enlarged model space of RPA states in the present work, and the slightly different evaluation radii ($R = 7.6\text{ fm}$ versus 7.2 fm).

The most significant advance of the present work for ^{208}Pb is the determination of β_v at $R = 5.7\text{ fm}$, which was inaccessible in Ref. [34]. As shown in Table I and Fig. 5, β_v exhibits a pronounced energy dependence, decreasing from 1.71 fm at $E = 7\text{ MeV}$ to 1.37 fm at $E = 30\text{ MeV}$, a variation of approximately 20%. The enhanced volume nonlocality at low energies is consistent with a coherent coupling to the low-lying collective excitations of this doubly magic nucleus, an interpretation supported by the multipole decomposition

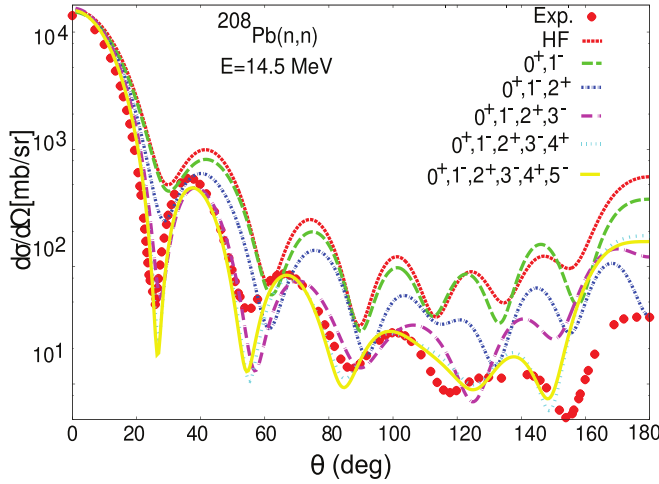


FIG. 6. Contributions of the multipole modes to the angular distribution of neutron elastic scattering by ^{208}Pb at incident energy $E = 14.5$ MeV. The angular distributions are computed by solving the nonlocal Schrödinger equation exactly with the DWBA98 code [45], without introducing any local-equivalent (Perey-type) approximation. The solid line denotes the HF result. The short-dashed, dotted, long-dashed, dotted/short-dashed, and dotted/long-dashed lines correspond to calculations with maximum multipolarity $L_{\text{max}} = 1, 2, 3, 4$, and 5 , respectively, and the points are the experimental data. Adapted from Ref. [38].

of Fig. 6, which shows that the absorption is built mainly from the most collective 3^- , 2^+ , and 4^+ modes. The interior absorptive strength that makes β_v accessible (absent in the more restricted SIII calculation of Ref. [34]) originates from the fully self-consistent SLy5 residual interaction and the complete natural-parity RPA model space employed here [35,36,38–40]. Within this interpretation, the interior coupling is strongest at low incident energy, where the superposition of the dominant collective modes produces a broad nonlocal range extending deep into the nuclear interior and hence a large β_v ; as the incident energy increases and more particle-hole configurations become accessible, this interior coupling is progressively diluted, the interior nonlocality is suppressed while the surface nonlocality remains comparatively stable, and the two curves eventually cross. This behavior could not have been anticipated from the surface-only analysis of Ref. [34], and shows why a description of the absorptive potential over the entire nuclear volume is needed.

Several systematic trends emerge from this comprehensive analysis. First, at higher energies ($E \gtrsim 20$ – 25 MeV) we find $\beta_s \gtrsim \beta_v$ for all targets. We interpret this ordering as reflecting the surface-peaked character of the transition densities of the dominant collective modes [34], which localizes the particle-vibration coupling in the surface region where the density gradient is largest. We emphasize, however, that this ordering is *not* universal: for the heavier and neutron-rich systems (^{48}Ca and ^{208}Pb) it is inverted at low energy, where the coherent coupling to interior collective strength makes

$\beta_v > \beta_s$. Rather than a fixed hierarchy, the present results thus reveal an energy-dependent competition between surface- and volume-localized nonlocality. Second, the magnitude of $\Delta\beta$ at intermediate energies varies systematically across the nuclear chart: it is largest for ^{16}O (≈ 0.24 – 0.72 fm), decreases for ^{40}Ca (≈ 0.02 – 0.24 fm) and ^{48}Ca (≈ 0.07 – 0.15 fm), and shows a complex energy-dependent pattern for ^{208}Pb (from ≈ -0.38 fm at $E = 5.5$ MeV to $\approx +0.16$ fm at high energies), reflecting the interplay between shell structure, neutron excess, and nuclear density profile. Third, both β_v and β_s generally decrease with increasing incident energy, which can be understood as a consequence of the diminishing relative importance of low-lying collective excitations at higher energies: as more particle-hole configurations become energetically accessible, the coherent coupling to collective vibrations, which is the primary source of long-range nonlocality, is progressively diluted. This effect is most dramatic for β_v in ^{208}Pb , where the variation reaches $\approx 32.5\%$ from the peak value to the highest energy considered. Finally, for all targets and energies considered, the obtained β values lie in the range 1.01–2.03 fm.

IV. SUMMARY AND CONCLUSIONS

We have studied the nonlocality of the imaginary part of a microscopic optical potential that links the structure of the target to the reaction dynamics at low energies, for neutron elastic scattering on ^{16}O , ^{40}Ca , ^{48}Ca , and ^{208}Pb . Instead of a constant value fitted to experimental data, the extracted nonlocality parameter is a radius- and energy-dependent function $\beta(R, E)$, whose values in both the volume and surface regions lie between 1.01 and 2.03 fm. Although the present approach is constrained by the use of zero-range Skyrme interactions, the results obtained provide meaningful insights about the nonlocality properties that may be useful for the development of next-generation optical potentials applicable across the nuclear chart, from light to heavy systems.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

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