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LESSON PLANNING AS A REFLECTION OF PRE-SERVICE MATHEMATICS TEACHERS' MODELING TEACHING COMPETENCE

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This study developed a teacher training program that integrates mathematical modeling and examined its impact on pre-service mathematics teachers' modeling teaching competence through their lesson plans. Based on the literature, a 10-criteria rubric was designed and applied to analyze 20 lesson plans from pre-service math teachers. The results revealed both strengths and weaknesses in their instructional design. The rubric proved to be a valuable tool for both research and practice-oriented reflection. Several recommendations for the teacher training program were proposed to enhance pre-service teachers' competence in teaching mathematical modeling.

Keywords: mathematical modeling, modeling teaching competence, lesson plan, pre-service mathematics teachers.

Introduction

In recent decades, the integration of mathematical modeling into school curricula has gained increasing support as a means to enhance mathematics education in a more practical and applicable way (Kaiser, 2020). Recognizing the importance of connecting learning with real-world contexts, Vietnam's 2018 education reform emphasized that mathematics instruction should go beyond knowledge transmission and focus on developing students' ability to apply mathematics into real-life problem-solving. As part of this reform, modeling competence has been identified as one of the five core mathematical competencies that students must develop in school. This shift presents new challenges for teacher education, particularly in preparing future mathematics teachers to effectively incorporate modeling into their teaching practices (Ferri & Blum, 2009).

A key aspect of effective teaching is lesson planning, which is not only a routine responsibility but also a critical indicator of professional competence. Lesson plans reflect a teacher's ability to structure instruction, design meaningful learning experiences, and implement pedagogical content knowledge in practice (Tillema, 2009; Kitsantas & Baylor, 2001; Baylor, 2002; Yildirim, 2003). However, research on lesson planning as a specific skill among pre-service teachers remains limited (König et al., 2020). This study examines how mathematics pre-service teachers apply their peda-

gogical content knowledge of modeling to design lesson plans after participating in a training program. Furthermore, a specialized analytical tool has been developed to assess the quality of lesson plans focused on modeling-based instruction.

Theoretical Framework

Pedagogical Content Knowledge to Teach Modeling

Modeling is the process of using mathematics to solve real-world problems, in order to gain a deeper understanding of a situation, explore potential solutions, explain phenomena, and support decision-making (Niss et al., 2007). Researchers have represented the modeling process through various cycles, but common stages typically include analyzing and reconstructing the real-world situation, selecting an appropriate mathematical model, working within a mathematical environment, interpreting and evaluating mathematical results in context, refining the model, and iterating the process until a satisfactory solution is achieved.

Teacher knowledge is a key component of professional competence and has been a significant focus of research in recent decades (Baumert et al., 2010). It encompasses various domains, including content knowledge, pedagogical knowledge, and pedagogical content knowledge. Pedagogical content knowledge, in particular, refers to the use of illustrations, examples, explanations, and instructional strategies that help students overcome difficulties and misconceptions, making learning more effective (Shulman, 1987). In other words, pedagogical content knowledge includes the knowledge teachers need to design lesson plans, interact effectively in instructional settings, and understand curriculum structures (Kaiser et al., 2017). Research indicates that teachers with strong pedagogical content knowledge are better equipped to engage students cognitively and enhance their learning experiences (Kunter et al., 2013).

The concept of pedagogical content knowledge highlights the importance of providing pre-service teachers with opportunities to experience mathematical modeling as active learners while equipping them with the necessary knowledge and skills to teach modeling through structured learning tasks. Ferri (2018) identified four key aspects of pedagogical content knowledge essential for teaching modeling:

- Knowledge of the modeling cycle, its role in teaching mathematics, the criteria of the modeling task.
- Knowledge and skills to solve modeling tasks, and anticipate potential difficulties students may face during the process.
- Knowledge and skills in planning lessons that incorporate modeling and providing appropriate guidance and support throughout the modeling process.
- Knowledge and skills to diagnose students' difficulties in different stages of the modeling cycle, as well as evaluate students' performance on modeling tasks.

By fostering these competencies, teacher education programs can better prepare pre-service teachers to integrate modeling effectively into their instructional practices.

Lesson Planning – a Part of Teacher's Professional Competence

The primary responsibility of teachers is instruction, making the development of their professional competence synonymous with enhancing their teaching skills (Kaiser et al., 2017). Teacher training programs often emphasize lesson planning to help pre-service teachers reflect on what to teach, how to teach, and how to assess student learning. Effective lesson planning also allows them to

anticipate potential challenges in the classroom and prepare appropriate support strategies (Baylor, 2002; Yildirim, 2003). Ruys et al. (2012) demonstrated that well-structured lesson plans not only positively impact student achievement by creating more learning opportunities but also enhance the quality of teachers' instructional practices. Tillema (2009) suggests that analyzing lesson plans is a valuable approach for gaining insights into teachers' professional competence. However, empirical studies on lesson planning skills and the assessment of lesson plans among mathematics education students remain limited (König et al., 2020).

During the lesson planning process, teachers need consider several key elements, including learning content, lesson objectives, student characteristics, teaching and assessment methods, and appropriate instructional resources (Mutton et al., 2011). One of the most critical aspects of lesson planning is selecting and designing learning tasks that structure student activities in the classroom, as these tasks integrate various instructional decisions (Kang, 2017). The learning tasks chosen for a lesson reflect how its objectives are defined—what knowledge, skills, or competencies students are expected to develop. Moreover, these tasks should be designed to stimulate student cognition (Neubrand et al., 2013). The way teachers incorporate learning tasks into lesson plans provides valuable insight into their pedagogical flexibility and adaptability.

Methodology

The study was conducted over three semesters, from February 2020 to February 2021, involving 80 third-year students from the Mathematics Department at Hue University of Education, Vietnam. We integrated Ferri's (2018) four aspects of pedagogical content knowledge for teaching mathematical modeling into four math teaching methods courses in the third and fourth years of the teacher training program. Specifically, students engaged in examining real-world problems in textbooks from various countries, understanding the modeling cycle, analyzing and solving modeling tasks, designing modeling tasks, planning integrated modeling lessons, conducting modeling projects, and assessing modeling products.

Data collection. In February 2021, students worked in groups of four to develop lesson plans that integrated mathematical modeling, either to introduce new concepts or to apply previously learned knowledge. Each group selected a high school-level lesson and incorporated pedagogical content knowledge for teaching modeling into their instruction. A total of 20 lesson plans were collected during this phase.

Data analysis. Based on the comprehensive literature review on the guidelines for teaching mathematical modeling proposed by Ferri (2018), we developed an assessment scale to evaluate pre-service teachers' ability to integrate modeling into their classrooms through their lesson plans. This process consisted of three stages: (1) Identifying key aspects - We determined the factors influencing the effectiveness of modeling instruction to establish evaluation criteria, including objectives (C7), task design (C1, C2, C3), the modeling process (C4, C10), diagnosis and guidance strategies (C5, C6), the organization of implementation (C8, C9); (2) Expert review and refinement - Four researchers with expertise in mathematical modeling and teacher education reviewed the content for validity and suggested revisions, leading to a near-final version; (3) Pilot analysis and final adjustments: Two researchers used the final version to analyze two sample lesson plans, discussing their scoring process and justifications. Minor refinements were made to the criteria descriptions, making them more specific and precise.

The final assessment scale consists of ten criteria (C):

- (C1) The modeling problem aligns with students' mathematical knowledge, allowing them to solve it using their prior understanding.
- (C2) The context is engaging, realistic, understandable, and relatable to students.
- (C3) The problem is authentic and clearly stated.
- (C4) The problem-solving process follows all stages of the modeling cycle.
- (C5) Anticipation of possible solution approaches and assessment strategies.
- (C6) Preparation of appropriate intervention and guidance strategies at different levels.
- (C7) The lesson achieves its intended objectives, such as mathematical understanding, applying mathematics, or improving specific modeling competencies.
- (C8) Adequate time is allocated for students to engage in problem-solving.
- (C9) The implementation is explicitly described.
- (C10) The lesson encourages students' metacognitive activities, such as reflection, validation, and exploring alternative solutions.

Evaluation is based on the presence and clarity of each criterion. Each criterion is scored on a 0–2 scale (0 = Not present; 1 = Needs improvement; 2 = Meets expectations)

This assessment framework provides a structured approach to evaluating pre-service teachers' competency in designing modeling-based lessons, ensuring alignment with effective instructional practices.

Results

Using the assessment scale for integrating mathematical modeling into teaching, as described earlier, we coded the student groups' lesson plans into three levels: 0, 1, and 2. The results are presented in Figure 1.

Based on the statistical results, all student groups scored above the average level for integrating mathematical modeling into teaching, with total scores of 10 or higher. Specifically, five groups achieved scores between 15 and 18 (out of 20), while the remaining 15 groups scored between 10 and 14. Additionally, students performed exceptionally well in criteria 1, 7, 5, and 8. However, their scores were notably low in criteria 10 and 6. For all other criteria, students achieved above-average performance. In the following section, we will analyze each criterion in detail, examining how student groups demonstrated their ability to integrate mathematical modeling into teaching through their lesson plans.

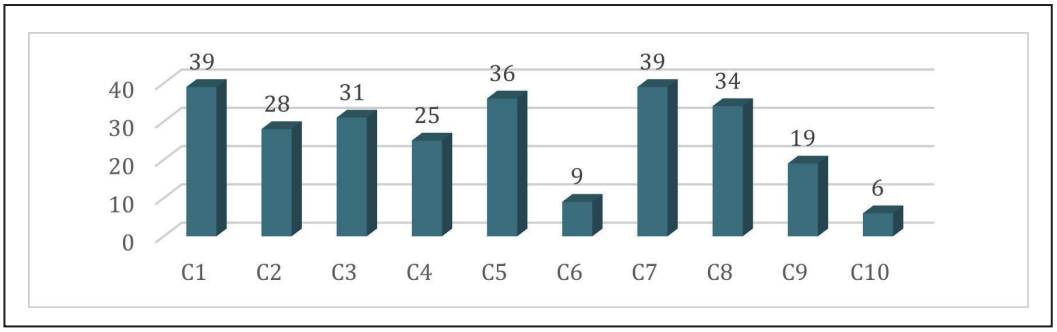

Figure 1. Scores for integrating mathematical modeling into teaching

Most student groups met Criterion 1 by proposing modeling problems aligned with students' mathematical knowledge and solvable using prior knowledge, except for Group 3, where the problem was more suitable for 8th-grade rather than 11th-grade students. For Criterion 2, all groups created engaging and relatable contexts, but only 8 out of 20 achieved the highest score for real-world relevance, while 12 groups scored lower due to unrealistic scenarios. For example, Group 3 overlooked door areas when calculating the painted wall area, and Group 5 framed the problem as a word problem, making it easier for learners to detach from the real-world context. In Criterion 3, 9 out of 20 groups scored only 1 point due to unrealistic, unclear problem statements, or reliance on mathematical rather than everyday language. For Criterion 4, only 5 out of 20 groups guided students through the entire modeling process, with teachers handling the model-building step, while the remaining 15 groups either started with a given mathematical model or omitted student reflection on mathematical results. In Criterion 5, 16 out of 20 groups earned full marks for proposing appropriate solutions, while Groups 3, 5, 10, and 14 scored lower due to incomplete or vague approaches. Most groups presented only one solution, except for Group 12, which proposed three. For Criterion 6, only Group 12 received full marks by providing structured guidance using thought-provoking questions, anticipating student responses to support flexible learning, while 12 groups scored 0 for lacking student support during problem-solving. In Criterion 7, most groups met the instructional goal of applying or introducing new knowledge, except for Group 3, which scored 1 point due to unrelated lesson content. Among the 19 groups that earned full marks, 5 used modeling to teach new concepts, while 14 applied it to real-world problem-solving, reinforcing the relevance of mathematics. For Criteria 8 and 9, most groups implemented modeling activities through group work, but some presented problems as word problems, causing students to disengage from real-world contexts and affecting time allocation. Group organization was often generic, lacking optimal group sizes for effective collaboration. In Criterion 10, no group provided questions or guidance for students to evaluate the accuracy or efficiency of their modeling process or explore alternative solutions, though six groups (1, 2, 3, 4, 12, and 13) considered the real-world validity of their results.

Conclusion

This study developed an assessment tool to analyze pre-service teachers' lesson plans on related to teaching mathematical modeling. This tool can also serve as a valuable resource for identifying strengths and weaknesses in modeling-based instruction within teacher training programs. The findings show that most pre-service teachers designed modeling problems aligned with students' knowledge and proposed appropriate problem-solving approaches, achieving the intended learning goals.

However, some challenges remain. Many struggled to create authentic modeling scenarios, often using unrealistic or overly mathematical problems. They also tended to guide students through only parts of the modeling process, lacked strategies to support students in overcoming difficulties, and gave little attention to student reflection on results. Addressing these issues in teacher training can enhance their ability to teach modeling effectively.

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