ICMI-EARCOME8

Taipei, Taiwan, May 7-11, 2018

Proceedings

of the 8th ICMI-East Asia Regional Conference on Mathematics Education

EARCOME8
TAIWAN / 2018

Editor: Feng-Jui Hsieh

Volume 2

Organizers

Taiwan Association for Mathematics Education

Department of Mathematics National Taiwan Normal University

Shi-da Institute for Mathematics Education

Flexibility in Mathematics Education

Taipei, Taiwan May 7-11, 2018

Cite as:
Hsieh, F.-J. (Ed.), (2018). *Proceedings of the 8th ICMI-East Asia Regional Conference on Mathematics Education*, Vol 2, Taipei, Taiwan: EARCOME.

Website: **http://earcome8.math.ntnu.edu.tw**

The proceedings are also available online

ISBN 978-986-05-5784-8

Cover design: Chia-Hsin Lin

VIETNAMESE PROSPECTIVE MATHEMATICS TEACHERS' MATHEMATICAL KNOWLEDGE FOR TEACHING THE DERIVATIVE AND IMPLICATIONS FOR TEACHER PREPARATION PROGRAMS

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Introduction

Preparing future mathematics teachers with the appropriate mathematical knowledge necessary for their effective teaching is an important issue in mathematics teacher education. There have been several studies conducted in an attempt to identify what kinds of knowledge teachers need to teach mathematics effectively (Shulman, 1986; Ball, Thames, & Phelps, 2008; Hill, Ball, & Schilling, 2008). From the work of Shulman (1986), many researchers have tried to develop and clarify the nature of the different types of knowledge, especially the pedagogical content knowledge (PCK) that teachers need to teach mathematics effectively. Ball and her colleagues made an important contribution by developing a model of mathematical knowledge for teaching (the model MKT) for assessing and developing the domains of teachers' knowledge for teaching.

Many researchers have used or adapted the model MKT for studying prospective mathematics teachers' knowledge for teaching specific topics. However, there have been very few studies focusing on the exploration of prospective mathematics teachers' mathematical knowledge of the derivative. Recently, Pino-Fan, Godino & Font (2018) assessed key epistemic features of didactic-mathematical knowledge of prospective mathematics teachers (PMTs) about the derivative, but this research used a theoretical model of mathematical knowledge for teaching based on an onto-semiotic approach. In Vietnam, until now there has been no study on measuring and developing Vietnamese prospective mathematics teachers' knowledge for teaching.

In this study, we assess and develop Vietnamese PMTs' mathematical knowledge for teaching the derivative. More specifically, we aim at analysing the characteristics of Vietnamese prospective secondary mathematics teachers' MKT for teaching different meanings of the derivative. Finally, implications for the professional learning of Vietnamese PMTs to improve their mathematical knowledge for teaching the derivative are also discussed.

Theoretical Framework

Model of mathematical knowledge for teaching

The model MKT consists of two main domains of knowledge: subject content knowledge and pedagogical content knowledge. In each of these domains, the authors distinguished different types of knowledge. One of the special contributions of the model MKT is a distinct categorisation of different types of knowledge, and especially the existence of a specialized content knowledge (SCK) which is considered as knowledge unique to teaching.

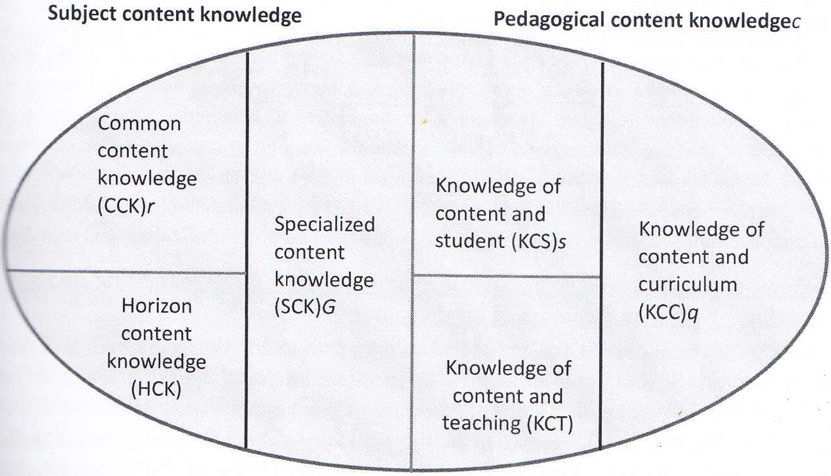


Figure 1. Model of mathematical knowledge for teaching (Ball, Thames, & Phelps, 2008)

Mathematical knowledge for teaching different meanings of the derivative

The derivative is an important concept in calculus. The history of the concept of derivative is closely linked to the problems of finding the tangent line to a curve and the instantaneous velocities in physics. The Vietnamese mathematics curriculum mentions explicitly both the geometric and physical meanings of the derivative. The analytical meaning (linear approximation) is implicitly presented through exercises.

<u>CCK: defining the concept of derivative and identifying different meanings of the derivative in different contexts</u>

Any user of mathematics should be able to identify the definition of derivative as well as to recognize the geometric, physical or analytical meanings of the derivative in a simple problem. For example, as a part of CCK, a user of mathematics should recognize the existence of the derivative of a function at a point based on its graphical representation (Pino-Fan, Godino & Font, 2018), or recognize that the instantaneous velocity of an object in motion means the derivative of the function characterising its motion at a point.

<u>SCK: ability to analyse and comment on right and wrong solutions given by students, to recognize the associations between different meanings of the derivative.</u>

In the context of teaching derivatives, as part of SCK, we focus on the ability of PMTs to analyse and comment on the accuracy of students' responses related to different meanings of the derivative. Another important aspect of SCK is the ability to perceive associations between different meanings of the derivative (Pino-Fan, Godino, & Font, 2018). A mathematical task that addresses the associations between these three meanings can be used to assess teachers' specialized content knowledge.

KCS: anticipating students' common answers, predicting and interpreting students' difficulties and misconceptions in the learning of derivatives

KCS is an important PCK for PMTs. According to Ball, Thames, & Phelps (2008), KCS is knowledge that combines knowing about students and knowing about mathematics and helps teachers predict student thinking. Teachers with this type of knowledge often have a deep understanding of students' thinking and of what makes the learning of mathematical concepts easy or difficult. In the context of teaching the different meanings of derivatives, as part of the KCS, we focus on examining the ability of PMTs to predict students' responses related to associations among the different meanings of the derivative, as well as to interpret students' difficulties and mistakes.

KCT: choosing and designing appropriate mathematical tasks for strengthening students' understanding of different meanings of the derivative

As part of KCT of the derivatives, a PMT should be able to choose mathematical tasks and design appropriate instructional strategies to facilitate students' learning about different meanings of the derivative. The current Vietnamese high school mathematics curriculum (MOET, 2008) pays a special attention to the geometrical and physical meanings of the derivative while the analytical meaning and the associations between different meanings of the derivative have not been emphasized.

KCC: recognizing the role of the derivative in school mathematics curriculum

KCC is knowledge that combines an awareness of a particular content of the curriculum and the teaching of that content. It is important for PMTs to know the role of the derivative in the school mathematics curriculum. In our study, we focus on the different meanings of the derivative.

HCK: identifying mathematical ideas related to the derivative

HCK is defined as awareness of how mathematical topics are related to each other in a curriculum, and of the connections among these topics. As part of the HCK of the derivative, PMTs should know mathematical ideas related to the derivative in order to solve a given problem. For example, the relationships among the derivative, the instantaneous velocity and the slope of the tangent of a curve are the mathematical ideas that can be used to solve a problem related to the physical meaning of the derivative.

Research Design

Context

This research can be considered as the first study of the assessment and development of PMTs' mathematical knowledge for teaching in Vietnam. Participants of the study are 70 PMTs studying at three different universities in Vietnam (a four year program). These PMTs had studied differential calculus in the first three semesters of their undergraduate program, and they had subsequently completed other courses related to mathematical analysis. They had also studied the subjects related to the teaching of mathematics.

Questionnaires and interviews

The research instrument was a questionnaire that consisted of three problems related to the geometric, physical and analytical meanings of the derivative. Each problem included six tasks, designed in accordance with the model MKT for assessing teachers' knowledge for teaching. In each of these three problems, each task referred to a type of knowledge among six different types of teachers' knowledge of the model MKT, with a focus on the CCK, SCK, KCS and KCT. In addition, an interview with some PMTs was done after they had completed the questionnaire.

Due to the limited scope of the paper, we only introduce here one of the three problems given in the questionnaire. This problem mentions the geometric meaning of the derivative.

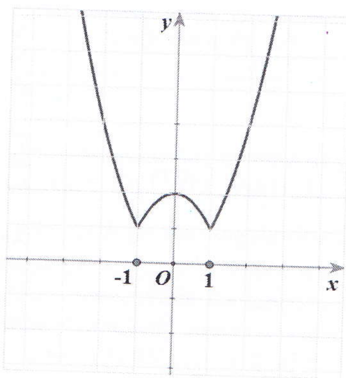
A problem

Below is the graph of a function $y = f(x)$ defined on an interval $(a;b)$:

i) Based on the graph of function, give comments on the existence of the derivative of the function at the points $x=-1, x=0$, and $x=1$.

ii) Assume that there are three students who propose the following comments:

Student A: The function is differentiable at the points $x=-1$ and $x=1$ because its graph is a continuous curve. In addition, $f'(-1), f'(0)$ and $f'(1)$ are strictly positive since the graph of function is above the x-axis.



Student B: The function is not differentiable at $x=-1$ and $x=1$ since its graph seems to be "broken" at these two points.

Student C: The function is differentiable at $x=-1$ and $x=1$ since its graph has the tangent lines on the left and right sides at the points $(-1;1)$ and $(1;1)$.

Please give comments on these three students' answers, analyse the accuracy of these answers and explain.

iii) 1. At what points is the instantaneous rate of change of y with respect to x equal to zero? Explain.

2. Please anticipate students' common responses (right or wrong) to the above

question and students' difficulties on this task. Give the most reasons for these students' common responses and difficulties.

iv) Assume that you aim to strengthen students' understanding of the geometric meaning of the derivative. What problem will you give to students? Explain why this problem is appropriate for the above goal?

v) In your opinion, this problem could be used to measure what content area in the curriculum?

vi) In your opinion, what mathematical ideas could be used to answer the above questions?

Figure 2. A problem from the questionnaire

Based on the definition of each type of knowledge of the model MKT, we classified six tasks in the above problem as shown in the following table:

Table 1. Components of MKT elicited by each of the questions given in the problem 1

Types of knowledge	Tasks	Indicators
CCK	i)	Defining the concept of derivative and identifying different meanings of the derivative in different contexts.
SCK	ii)	Ability to analyze and comment on right and wrong solutions given by students
	iii.1)	Ability to recognize the associations between different meanings of the derivative.
KCS	iii.2)	Anticipating students' common answers, predicting and interpreting students'difficulties and misconceptions
KCT	iv)	Choosing and designing appropriate mathematical tasks for strengthening students' understanding of different meanings of the derivative.
KCC	v)	Recognizing the role of the derivative in school mathematics curriculum.

HCK	vi)	Identifying mathematical ideas related to the derivative.

These six tasks were coded by the two authors of this paper. In the following table, we illustrated the coding of the task iii.1). Other tasks were coded by using the similar technique.

Table 2. Illustration of the coding of the task iii.1)

Codes	Description
4	Giving correct answers and explanations (the instantaneous rate of change of y with respect to x is equal to zero at the point $x=0$)
3	Giving correct answers and explanations (the instantaneous rate of change of y with respect to x is equal to zero at the point $x=0$), but there are still some inaccurate arguments.
2	Giving an correct result (the instantaneous rate of change of y with respect to x is equal to zero at the point $x=0$), but not having provided explanations or giving an incorrect explanation.
1	Not having given a correct response, but having some adequate ideas for finding an explanation.
0	Not having given any response or giving an incorrect response to the task.

Results

The results of teachers' knowledge are presented in terms of six types of knowledge of the model MKT, that is, CCK, SCK, KCS, KCT, KCC and HCK.

Prospective mathematics teachers' CCK for teaching the derivative

As seen in Table 3, for the task i), 45.72 % of PMTs were able to give partially right comments on the existence of the derivative of the function at the points $x=-1, x=0$ and $x=1$. The number of PMTs were able to fully give exact comments and explanation is very small (5.72%). Especially, there were 22.85% (code 0) of the PMTs who did not give any response to this task or offered incorrect explanations. This means that many PMTs haven't acquired a robust understanding of the geometric meaning of the derivative.

Table 3. Codes regarding the PMTs' written explanations for the task i)

Codes	Illustrative example	Frequency	%
4	The graph of function is "broken" at the points $x=-1$ and $x=1$, the function is not differentiable at these points since there is no tangent line. The function is differentiable at $x=0$ since we can draw a tangent line at this point.	4	

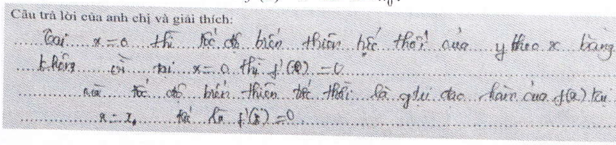
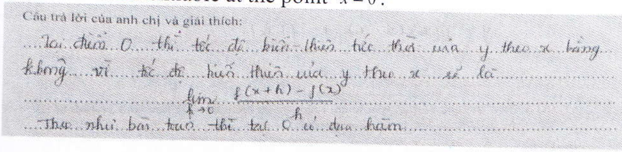
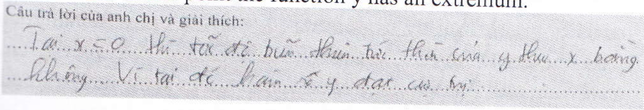
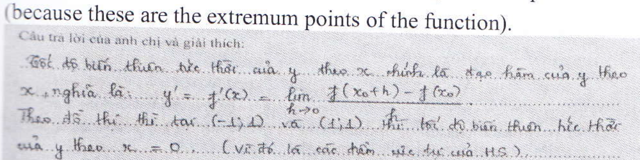
	Tại $x = -1$ và $x = 1$ thì đồ thị hàm số bị gấp khúc nên không tồn tại đạo hàm vì tại đó không vẽ được tiếp tuyến. Tại $x = 0$ có đạo hàm vì tại đó vẽ được tiếp tuyến.	
3	Since the function is "broken" at the points $x=1$ and $x=-1$, it is not differentiable at $x=1$ and $x=-1$. The function is differentiable at $x=0$. Hàm số bị gãy tại $x = 1$ và $x = -1$ nên hàm số không có đạo hàm tại các điểm $x = 1$ và $x = -1$. Hàm số có đạo hàm tại $x = 0$.	32
2	Based on the function graph, we can rewrite the function as $f(x)=x^2$ if $x\in(-\infty;-1]\cup[1;+\infty)$ and $f(x)=-x^2+2$ if $x\in(-1;1)$. The function $f(x)$ is continuous at $x=-1$, $x=0$ and $x=1$ but the function is differentiable at $x=0$ and it is not differentiable at $x=-1$ and $x=1$. Dựa vào đồ thị hàm số đã cho, ta có thể viết được hàm số như sau: $y = f(x) = \begin{cases} x^2 \text{ nếu } x \in (-\infty;-1]\cup[1;+\infty). \\ -x^2+2 \text{ nếu } -1 < x < 1. \end{cases}$ Hàm số $f(x)$ liên tục tại $x = -1; x = 0$ và $x = 1$ nhưng tại $x = 0$, hàm số có đạo hàm và tại $x = \pm 1$, hàm số không có đạo hàm tại đó.	17
1	The derivatives of the function at the points $x=-1$ and $x=0$ are equal to zero because the function admits the minimum values at the points $x=-1, x=1$ and $x=0$. Đạo hàm của hàm số tại các điểm $x = -1; x = 0$ và $x = 1$ bằng 0 vì hàm số đạt cực tiểu tại $x = \pm 1$ và cực đại $x = 0$	1
0	The derivative of the function at each point $x=-1$, $x=0$ is either equal to zero or undefined. Đạo hàm của hàm số tại các điểm $x = -1; x = 0; x = 1$ bằng 0 hoặc không xác định	16

Prospective mathematics teachers' SCK for teaching the derivative

Table 4 shows whether or not the PMTs were able to identify the association between the physical meaning (instantaneous rates of change) and the geometric meaning of the derivative (smooth or non-smooth curves). As seen in Table 4, more than half of the PMTs (48.58 %+ 7.14 % = 55.72 %) were not able to give a correct answer to the task iii.1) or offered an incorrect explanation. Only five (7.14 %) of the PMTs offered exact solutions and explanations for this task. It can be stated that most of the PMTs haven't adequate SCK of the association between the physical and geometric meanings of the derivative.

For each of the other four types of teachers' knowledge (KCS, KCT, KCC, HCK), we also made a table categorising students' written responses according to the given codes. Due to the limited scope of the paper, we only summarize some main results related to the students' KCS and KCT.

Table 4. Codes regarding the PMTs' written explanations for the task iii.1)

Codes	Illustrative example	Frequency	%
4	The instantaneous rate of change of y with respect to x is equal to zero at the point $x=0$ since $f'(x)=0$ at $x=0$. The instantaneous rate of change is the derivative of $f(x)=0$ at $x=x_0$.  Câu trả lời của anh chị và giải thích: Tại x = 0 thì tốc độ biến thiên tức thời của y theo x bằng không vì tại x = 0 thì f'(x) = 0 mà tốc độ biến thiên tức thời là giá trị đạo hàm của f(x) tại x = x₀ thì là f'(x) = 0	5	
3	At the point $x=0$ the instantaneous rate of change of y with respect to x is equal to zero since the instantaneous rate of change of y with respect to x is $\lim\limits_{h \to 0} \frac{f(x+h)-f(x)}{h}$. According to the problem, the function is differentiable at the point $x=0$.  Câu trả lời của anh chị và giải thích: Tại điểm 0 thì tốc độ biến thiên tức thời của y theo x bằng không vì tốc độ biến thiên của y theo x sẽ là $\lim\limits_{h \to 0} \frac{f(x+h)-f(x)}{h}$ Theo như bài toán thì tại 0 có đạo hàm	19	
2	At $x=0$ the instantaneous rate of change of y with respect to x is equal to zero since at that point the function y has an extremum.  Câu trả lời của anh chị và giải thích: Tại x = 0 thì tốc độ biến thiên tức thời của y theo x bằng không. Vì tại đó hàm số y đạt cực trị.	7	
1	The instantaneous rate of change of y with respect to x is exactly the derivative of y with respect to x, that means $y' = f'(x) = \lim\limits_{h \to 0} \frac{f(x_0+h)-f(x_0)}{h}$. Based on the graph, at the points $(-1;1)$ and $(1;1)$ the instantaneous rate of change of y with respect to $x=0$ (because these are the extremum points of the function).  Câu trả lời của anh chị và giải thích: Tốc độ biến thiên tức thời của y theo x chính là đạo hàm của y theo x, nghĩa là: $y' = f'(x) = \lim\limits_{h \to 0} \frac{f(x_0+h)-f(x_0)}{h}$ Theo đồ thị thì tại (−1;1) và (1;1) thì tốc độ biến thiên tức thời của y theo x = 0 (vì đó là các điểm cực trị của HS).	5	

0	At the extremum points (smooth). Câu trả lời của anh chị và giải thích: Tại các điểm cực trị (trơn)	34

Regarding the prospective mathematics teachers' KCS, we found that there were few PMTs who could give adequate explanations for the three students' responses in the task ii). It can be stated that, for the most part, prospective mathematics teachers' KCS was not at an adequate level. Concerning the teachers' KCT, for the task iv), approximately half of PMTs were able to offer a type of problem that could be used to strengthen the geometric meaning of the derivative. Fifteen (21.42 %) of the PMTs offered at least one appropriate exercise with explanations that could be used to strengthen students' understanding of the geometric meaning of the derivative.

Conclusion and Discussion

In this study, we examined Vietnamese PMTs' knowledge for teaching the derivative in terms of six types of knowledge of the model MKT. As a result of the study, it was observed that many Vietnamese PMTs haven't acquired a good understanding of the geometric meaning of the derivative, although the derivative is an important topic of the high school mathematics curriculum. The findings also indicate that the prospective mathematics teachers' SCK was below the level expected for teaching aspects of the geometric meaning of the derivative. This result is similar to prospective teachers' KCS and KCT, in which they exhibited many difficulties and gaps in the understanding of students' learning of the derivative.

These findings provide us with first evidence of Vietnamese PMTs' inadequate understanding of mathematics for teaching the derivative. In particular, the prospective mathematics teachers' PCK is not at an adequate level. The primary results obtained from this study could be partially interpreted by examining the current professional development programs for PMTs in Vietnam. In fact, although Vietnamese PMTs take courses on mathematics education throughout their undergraduate education, these courses traditionally focus on rules, techniques and procedural knowledge for teaching mathematics. The aspects of the SCK, KCS and KCT or the epistemology of a mathematical concept seem to be less analysed in these courses. Therefore, we suggest that the content and duration of these courses on mathematics instruction should be reviewed and modified.

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