The Parametric Resonance of Confined Acoustic Phonons and Confined Optical Phonons by an External Electromagnetic Wave in Cylindrical Quantum Wires with an Infinite Potential

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Abstract— The parametric resonance of confined acoustic phonons and confined optical phonons by an external electromagnetic wave in cylindrical quantum wires with an infinite potential is studied by using a set of quantum kinetic equations for confined phonons. The analytical expression of the threshold amplitude $E_{threshold}$ of the field in a cylindrical quantum wires with an infinite potential is obtained. The formula of $E_{threshold}$ contains two quantum numbers (m, k) characterizing confined phonons. The dependence of the threshold amplitude $E_{threshold}$ on the temperature T of the system, the wave vector $\vec{q}_z$, the frequency Ω of an external electromagnetic waves and the radius of the wires R is studied. Numerical computations and graphs are performed for GaAs-GaAsAl cylindrical quantum wires. The results are compared with the case of unconfined phonons.

1. INTRODUCTION

In quantum wires (QW), the motion of electrons and phonons is restricted in two dimensions, so that can flow freely in one dimension. The confinement of electrons in these systems has changed the electrons mobility remarkably. This has resulted in a number of new phenomena, which concern a reduction of sample dimensions. Many attempts have conducted dealing with these behaviors, for examples: the problems of the linear absorption coefficient in QW [1]; the nonlinear absorption in rectangular QW [2], in QW [3] have been studied. Electron interaction with confined acoustic phonons [4], the polar-optic phonons and high field electron transport [5] and self-consistent electronic structure [6] have also been researched in cylindrical quantum wires (CQW). And parametric interactions and transformations (PIT) are the interesting problems in CQW.

As we know that the electron gas becomes non-stationary in the presence of an external electromagnetic wave (EEW). When the conditions of the parametric resonance are satisfied, PIT of the same kinds of excitations, such as phonon-phonon and plasmon-plasmon excitations, or of different kinds of excitations, such as plasmon-phonon excitations, will arise, i.e., the energy exchange processes between these excitations will occur [7, 8]. For semiconductor nanostructures, there have been several works on the generation and amplification of acoustic phonons [9–11]. The PIT of acoustic and optical phonons have been considered in bulk semiconductors [12–14], in low dimensional semiconductors [15, 16] and CQW [17]. However, the parametric resonance of acoustic and optical phonons by an EEW in CQW with an infinite potential in the case of confined phonons have not studied yet. Therefore, in this paper, we continue to study the parametric resonance of confined acoustic and confined optical phonons in CQW with an infinite potential by an EEW. Numerical calculations are carried out with a specific GaAs/GaAsAl CQW. This result has been compared with the case of unconfined phonons [17], which shows clearly the effect of confined phonons as in [18, 19].

2. THE PARAMETRIC RESONANCE OF CONFINED ACOUSTIC PHONONS AND CONFINED OPTICAL PHONONS BY AN EXTERNAL ELECTROMAGNETIC WAVE IN CQW

We use a simple model for a CQW, in which a one-dimensional electron, phonon gas is confined by the infinity potential $V(x, y)$ along the x-y direction. So electrons and phonons are free only on the z plane. A laser field $\vec{E} = \vec{E}_o \sin(\Omega t)$ irradiates the sample in a direction, which is normal to the z plane, its polarization is along the (x, y) axis and its strength is expressed as a vector potential $\vec{A}(t) = c\vec{E}_o \cos(\Omega t)/\Omega$. If the confined electron-confined acoustic phonons and confined optical phonons interaction potential is used, the Hamiltonian for the system of the confined electron and the confined acoustic and confined optical phonons is written as:

$$H = H_e + H_{aph} + H_{oph} + H_{e-aph} + H_{e-oph} \tag{1}$$

In order to establish a set of quantum kinetic equations for confined acoustic phonons and confined optical phonons, we use the general quantum distribution functions for the confined phonons [20] $\langle b_{m,k,\vec{q}_z}\rangle_t$ and $\langle c_{m,k,\vec{q}_z}\rangle_t$, where $\langle\psi\rangle_t$ denotes a statically, average at the moment $\langle\psi\rangle_t = Tr|\hat{W}\hat{\psi}\rangle$, ($\hat{W}$ is the density matrix operator); m, k are quantum number characterizing confined phonons. Using Hamiltonian in Equation (1) and realizing operator algebraic calculations, we obtain a set of coupled quantum transport equations.

Using Fourier transformation:

$$\left\langle\psi_{m,k,\vec{q}_z}\right\rangle_t = \frac{1}{2\pi}\int\limits_{-\infty}^{+\infty}\Psi_{m,k,\vec{q}_z}(\varpi)\,e^{-i\varpi t}d\varpi; \quad \Psi_{m,k,\vec{q}_z}(\varpi) = \int\limits_{-\infty}^{+\infty}\left\langle\psi_{m,k,\vec{q}_z}\right\rangle_t e^{i\varpi t}dt \tag{2}$$

to solve coupled quantum transport equations then we obtain the general dispersion equation for PIT of confined acoustic phonons and confined optical phonons in CQW:

$$\begin{aligned}
&\left(\varpi^2 - \varpi^2_{m,k,\vec{q}_z} - \frac{2}{\hbar}\sum_{\alpha,\alpha',m,k}\gamma^2\left|I_{1D}(\vec{q}_z)\right|^2\varpi_{m,k,\vec{q}_z}\Pi_0(\vec{q}_z,\varpi)\right)\\
&\left((\varpi - l\Omega)^2 - \nu^2_{m,k,\vec{q}_z} - \frac{2}{\hbar}\sum_{\alpha,\alpha',m,k}\beta^2\left|I_{1D}(\vec{q}_z)\right|^2\nu_{m,k,\vec{q}_z}\Pi_0(\vec{q}_z,\varpi - l\Omega)\right)\\
&= \frac{4}{\hbar^2}\sum_{\alpha,\alpha',m,k}\sum_{l=-\infty}^{+\infty}\left\{\gamma^2\beta^2\left|I_{1D}(\vec{q}_z)\right|^4\nu_{m,k,\vec{q}_z}\varpi_{m,k,\vec{q}_z}\Pi_l(\vec{q}_z,\varpi)\,\Pi_l(\vec{q}_z,\varpi - l\Omega)\right\}
\end{aligned} \tag{3}$$

where γ, β are the electron-phonon interaction constants; $|I_{1D}(\vec{q}_z)|$ is the electron form factor which can be written from [5, 21]; $\nu_{m,k,\vec{q}_z}$, $\varpi_{m,k,\vec{q}_z}$ are the frequency of confined phonons; α, α' characterizing the states of electron in the quantum wire before and after scattering with phonon.

Here:

$$\Pi_l(\vec{q}_z,\varpi) = \sum_{\nu=-\infty}^{\infty} J_\nu\left(\frac{\lambda}{\Omega}\right)J_{l+\nu}\left(\frac{\lambda}{\Omega}\right)\Gamma_{m,k,\vec{q}_z}(\varpi + \nu\Omega) \quad \text{with} \quad \lambda = \frac{e\vec{q}\vec{E}_o}{m^*\Omega^2} \tag{4}$$

$$\Gamma_{m,k,\vec{q}_z}(\varpi + l\Omega) = \sum_{\vec{k}_\perp}\frac{f_{\alpha'}\left(\vec{k}_z - \vec{q}_z\right) - f_\alpha\left(\vec{k}_z\right)}{\left[\varepsilon_\alpha\left(\vec{k}_z\right) - \varepsilon_{\alpha'}\left(\vec{k}_z - \vec{q}_z\right) - \hbar l\Omega - \hbar\varpi + i\hbar\delta\right]}, \tag{5}$$

If we write the dispersion relation of confined acoustic phonons and confined optical phonons:

$$\varpi_{m,k,\vec{q}_z} = \varpi_a + i\tau_a; \quad \nu_{m,k,\vec{q}_z} = \varpi_o + i\tau_o \tag{6}$$

$$\tau_a = -\frac{1}{\hbar}\sum_{\alpha,\alpha',m,k}\gamma^2\left|I_{1D}(\vec{q}_z)\right|^2\mathrm{Im}\Pi_0(\vec{q}_z,\varpi) \tag{7}$$

$$\tau_0 = -\frac{1}{\hbar}\sum_{\alpha,\alpha',m,k}\beta^2\left|I_{ID}(\vec{q}_z)\right|\mathrm{Im}\Pi_0(\vec{q}_z,\varpi - l\Omega) \tag{8}$$

We obtain the resonance phonon mode:

$$\varpi^{(\pm)}_{\pm} = \varpi_a + \frac{1}{2}\left\{(v_a \pm v_o)\,\Delta(q) - i\,(\tau_a + \tau_o) \pm \sqrt{\left[(v_a \pm v_o)\,\Delta(q) - i\,(\tau_a - \tau_o)\right]^2 \pm \Lambda^2}\right\}, \tag{9}$$

where $\Delta(q) = q - q_0$ being the wave number for which the resonance is satisfied, $v_a\,(v_0)$ is the group velocity of the acoustic (optical) phonon; ϖ_a is the renormalization (by the electron-phonon interaction) frequency of the acoustic phonon and:

$$\Lambda = \frac{2}{\hbar}\sum_{\substack{\alpha,\alpha' \\ m,k}} \gamma\beta \left|I_{1D}(\vec{q}_z)\right|^2 \Pi_l(\vec{q}_z, \varpi_{m,k,\vec{q}_z}) \tag{10}$$

In Equation (10), the signs ($\pm$) in the subscript of $\varpi_{\pm}^{(\pm)}$ correspond to the signs ($\pm$) in the front of the root and the signs ($\pm$) in subscript of $\varpi_{\pm}^{(\pm)}$ correspond to the other sign pairs. The signs depend on the resonant condition:

$$\lambda^2 > \frac{4\Omega^2 \mathrm{Im}\Gamma_{m,k,\vec{q}_z}\left(\varpi_{m,k,\vec{q}_z}\right) \mathrm{Im}\Gamma_{m,k,\vec{q}_z}\left(\nu_{m,k,\vec{q}_z}\right)}{\left[\mathrm{Re}\Gamma_{m,k,\vec{q}_z}\left(\varpi_{m,k,\vec{q}_z}\right)\right]^2}$$

For instance, the existence of a positive imaginary part of $\varpi_{\pm}^{(\pm)}$ implies a parametric amplication of the acoustic phonon. In such case that $\lambda \ll 1$, corresponding to the maximal resonance, we obtain:

$$F = \mathrm{Im}\omega_{+}^{-} = \mathrm{Im}\left\{\omega_a + \frac{1}{2}\left[-i\left(\tau_a + \tau_o\right) \pm \sqrt{\left(\tau_a + \tau_o\right)^2 + \Lambda^2}\right]\right\} \tag{11}$$

From Equation (11), the condition for the resonant acoustic phonon modes to have a positive imaginary part leads to $|\Lambda|^2 > 4\tau_a\tau_o$. Using this condition and Equations (1)–(11), we have found out the intensity of the threshold field $E_{threshold}$ for EEF:

$$E_{threshold} = \frac{2m^*\Omega}{e\sqrt{q_z^2 + q_{m,k}^2}} \times \frac{\sqrt{\xi\left(\varpi_{m,k,\vec{q}_z}\right) \cdot \xi\left(\nu_{m,k,\vec{q}_z}\right)}}{\sqrt{\left[\theta\left(\varpi_{m,k,q_z}\right) - \theta(\varpi_{m,k,\vec{q}_z} - \Omega)\right]^2 + \left[\xi\left(\varpi_{m,k,\vec{q}_z}\right) - \xi(\varpi_{m,k,\vec{q}_z} - \Omega)\right]^2}} \tag{12}$$

where:

$$\begin{aligned}
\xi(\varpi_{m,k,\vec{q}_z}) &= \frac{m^*}{2\hbar^2\sqrt{q_z^2 + q_{m,k}^2}} \exp\left(\frac{1}{k_BT}\left(\varepsilon_F - \frac{\hbar^2 B_\alpha^2}{2m^*R^2}\right)\right) \exp\left(-\frac{m^*\varepsilon_{\alpha\alpha'}^2\left(\nu_{m,k,\vec{q}_z} - \Omega\right)}{2k_BT\hbar^2\left(q_z^2 + q_{m,k}^2\right)}\right) \\
&\quad \left\{1 - \exp\left(\frac{\hbar\left(\varpi_{m,k,\vec{q}_z} - \Omega\right)}{k_BT}\right)\right\} \\
\theta(\varpi_{m,k,\vec{q}_z}) &= \frac{\sqrt{2m^*\pi k_BT}}{2\hbar\pi} \times \frac{1}{\varepsilon_{\alpha,\alpha'}\left(\varpi_{m,k,\vec{q}_z}\right)}\left[\exp\left(\frac{\varepsilon_F}{k_BT}\right)\left(\exp\left(-\frac{1}{k_BT}\left(\frac{\hbar^2 B_\alpha^2}{2m^*R^2}\right)\right)\right.\right. \\
&\quad \left.\left. - \exp\left(-\frac{1}{k_BT}\left(\frac{\hbar^2 B_{\alpha'}^2}{2m^*R^2}\right)\right)\right)\right] \\
\varepsilon_{\alpha\alpha'}(\varpi_{m,k,\vec{q}_z}) &= \frac{\hbar^2 B_\alpha^2}{2m^*R^2} - \frac{\hbar^2 B_{\alpha'}^2}{2m^*R^2} - \frac{\hbar^2\left(q_z^2 + q_{m,k}^2\right)}{2m^*} - \hbar\varpi_{m,k,\vec{q}_z}
\end{aligned}$$

Here k_B is Boltzmann constant, $q_{m,k}^2$ can be written from [21], B_α is corresponding to the equation $J_n(B_\alpha) = 0$, ε_F is the Fermi energy and R is the radius of the wires respectively.

In Equation (12), we can see the marked difference between the case of confined phonons and unconfined phonons, the formula of $E_{threshold}$ contains a quantum number (m, k) characterizing confined phonons.

3. NUMERICAL RESULTS AND DISCUSSIONS

In order to clarify the mechanism for parametric resonance of acoustic-optical phonons in case of the confined phonons, in this section, we consider a $GaAs/GaAsAl$ CQW. The parametric used in the calculation are following [9, 18]: $m^* = 0.066m_o$, m_o being the mass of free electron; $\Omega = 8 \cdot 10^8$ Hz, $\hbar v_{\vec{q}_\perp}^m \approx \hbar v_0 = 36.25$ meV; $\omega_{\vec{q}_\perp}^m \approx v_a = 5370\,\mathrm{ms}^{-1}$, $k_B = 1.3807 \times 10^{-23}\,\mathrm{JK}^{-1}$; $e = 1.60219 \times 10^{-19}$ C; $\hbar = 1.05459 \times 10^{-34}\,\mathrm{Js}^{-1}$; $R = 5$ nm.

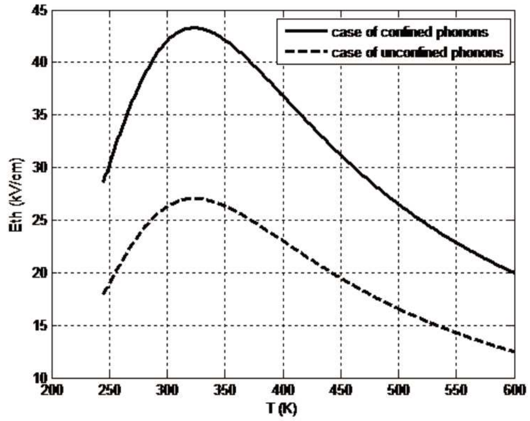

Figure 1: The dependence of $E_{threshold}$ on temperature T (K).

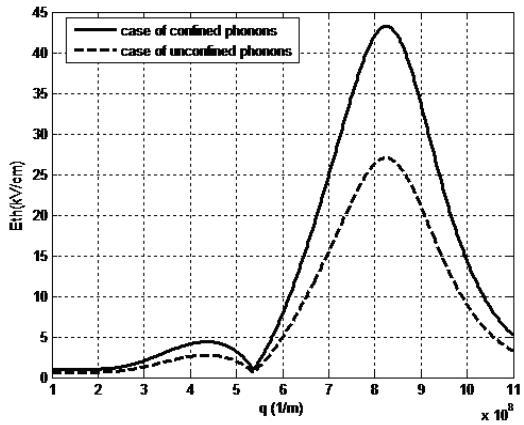

Figure 2: The dependence of $E_{threshold}$ on wave vector $\vec{q}_z$ (m^{-1}).

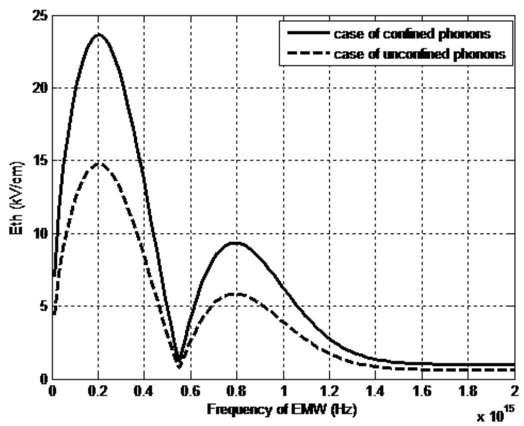

Figure 3: The dependence of $E_{threshold}$ on frequency Ω (Hz).

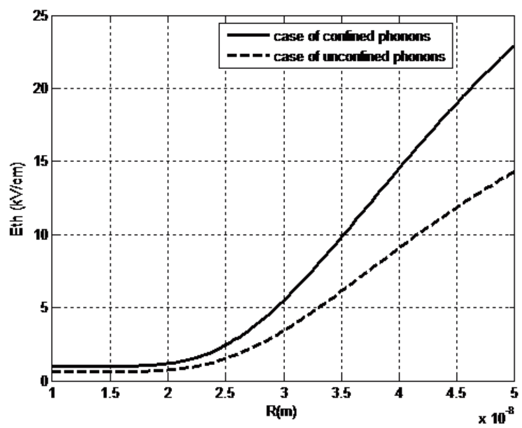

Figure 4: The dependence of $E_{threshold}$ on R (m).

In Fig. 1, It shows that $E_{threshold}$ as a function of temperature T in both cases of confined phonons and unconfined phonons. The graph shows that confined phonon increase the intensity of the threshold field $E_{threshold}$ in comparison with the case of unconfined phonons [17]. Namely, at the same temperature $T \sim 325\,\mathrm{K}$, $E_{threshold} \sim 43$ (kVcm^{-1}) in case of confined phonons, but $E_{threshold} \sim 22$ (kVcm^{-1}) in case of unconfined phonons.

In Fig. 2, present $E_{threshold}$ as a function of the wave vector at $T = 300\,\mathrm{K}$. The figure shows that the $E_{threshold}$ depends much strongly on wave vector that there are appearing two resonance peaks. Differing from the case of unconfined phonons [17], the curve has two lower resonance peaks. This is due to the fact that confined phonon has quantum wave number following the confined axis.

Figures 3 and 4 show that the strong dependences of $E_{threshold}$ on the frequency Ω and the radius of the wires are very clearly in case of confined phonons. There are two resonance peaks of $E_{threshold}$ at the difference values of frequency Ω in both cases of confined phonons and unconfined phonons. The $E_{threshold}$ increases fast following the radius of the wires and gets higher in case of confined phonons.

4. CONCLUSION

In this paper, we analytically investigated the possibility of parametric resonance of confined acoustic and confined optical phonons in CQW. We have obtained a set of quantum kinetic equations for transformation of phonons. However, the analytical solution applying to these equations can only be obtained with in some limitations. Using these limitations for simplicity we obtained the parametric resonant condition, the intensity of the threshold field $E_{threshold}$ for confined acoustic phonons and confined optical phonons in CQW. And we have also paid attention to $E_{threshold}$ in case of unconfined phonons to compare with the result above. We numerical calculated and graphed the intensity of the threshold field for $GaAs$/GaAs CQW. The results show that confined phonons cause some unusual effects. The threshold field $E_{threshold}$ depends strongly on the temperature T, the wave vector $\vec{q}_z$, the frequency Ω and the radius of the wires. Confined phonons will increase the values of the threshold field $E_{threshold}$.

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