Interference Cancellation in the Presence of Spread Angle-of-Arrival Using the Modified GSC Algorithm

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***Abstract*—The generalized sidelobe canceller (GSC) algorithm makes the optimum linearly constrained minimum variance (LCMV) beamformer much more efficient. However, when scattering the desired signal's direction of arrival (DOA) or scattering the interference's DOA, they cause severe degradation of the optimum solution. To improve this situation, at first, the general GSC algorithm with many constraints and the way to find the blocking matrix are showed in detail. Then, the modified constraint matrix with the new nominal angle constraint for scattering the desired signal's DOA is proposed. And the combination of the optimum weight vector and the window functions such as Hamming, Hanning, Kaiser, Chebyshev windows is used for scattering the interference's DOA. The good performance are evaluated by beam pattern and signal to interference plus noise ratio (SINR). Finally, numerical examples demonstrate that our proposals can enhance the performance of the GSC algorithm in the optimum beamformer.**

***Keywords*—Generalized sidelobe canceller (GSC), direction of arrival (DOA), beam pattern, signal to interference plus noise (SINR), statistically optimum beamformer.**

I. INTRODUCTION

Beamformers have important applications in fields such as radar, sonar, seismology, radio astronomy, medical imaging, microphone array speech processing, and wireless communications [1], [2], [3]. Beamformers can be classified according to data independent, satistically optimum, or adaptive , depending on how weights are chosen. The weights in a data independent beamformer do not depend on the array data and are chosen to present a specified response for all signal and interference scenarios. The weights in a statistically optimum beamformer are chosen based on the statistics of the array data to "optimize" the array response. Optimum beamforming requires some knowledge of the desired signal characteristics, either its statistics or its direction. The statistics of the array data are not usually known and may change over time so adaptive algorithms are typically employed to determine the weights. The adaptive algorithm is designed so the beamformer response converges to a statistically optimum solution.

The famous representative of statistically optimum beamformer is the linearly constrained minimum variance (LCMV) beamformer considered by Frost [4]. The basic idea of LCMV beamformer is to constrain the response of the beamformer so signals from the direction of interest are passed with specified gain and phase. The weight vector is calculated to minimize output variance or power subject to the response constraint. The generalized sidelobe canceller (GSC) proposed in [5] represents an alternative formulation of the LCMV problem. Essentially, the GSC is a mechanism for changing a constrained minimization problem into unconstrained form.

When there is a slight mismatch between the presumed and actual direction of arrival (DOA) of the desired signal, the statistically optimum LCMV beamformer has still high performance in interference suppression. Unfortunately, that is not true in case that the actual desired signal's DOA is scattered around the nominal angle that is completely different from presumed. The GSC tends to misinterpret the desired signal in input as interference and to suppress this component instead of maintaining distortionless response toward it. Especially, when the actual interference's DOA is scattered, the optimization of Wiener solution is no longer accurate. Thus, this may cause severe degradation of the optimum beamforming performance. Until recently, there are several approaches to robust adaptive beamforming based on GSC [6], [7], but there is no published work concerning the robustness of GSC optimum beamforming.

In this paper, we discuss the GSC algorithm on the statistically optimum LCMV beamformer and how to improve the performance of GSC when the signal's DOA is scattered. This paper is organized as follows. In section II, the signal model of a narrowband GSC beamformer and the model of scattering arrival angle are presented. In section III, the solution of GSC are described in detail and we add the method of finding the blocking matrix. In section IV, we propose how to improve GSC algorithm to robust against scattering the desired signal's DOA and the interference's DOA. Finally, simulation results of numerical examples and conclusions are given in section V and VI, respectively.

II. BACKGROUND

A. Signal Model

Consider a uniform linear array (ULA) of M omnidirectional antenna elements spaced by the distance d . Let a desired signal from far field impinge on the array from a *known* DOA θ_0 with $(L-1)$ uncorrelated interfering signals from *known* DOAs $\{\theta_1,\theta_2,...,\theta_{L-1}\}$, $(N-L+1)$ uncorrelated interfering signals from *unknow* DOAs $\{\theta_L,\theta_{L+1},...,\theta_N\}$, respectively. With the first element as the reference point, the $M\times 1$ signal's steering vector is given by

$$\mathbf{s}(\phi_k)=\left[1,\, e^{-j\phi_k},\, ...,\, e^{-j(M-1)\phi_k}\right]^T \qquad (0\le k\le N) \quad (1)$$

where $j=\sqrt{-1}$ and $\phi_k=(2\pi d/\lambda)\sin(\theta_k)$ is electrical angle according to DOA θ_k , in which λ is the signal wavelength. Then, the *n*th snapshot of the $M\times 1$ received equivalent baseband signal vector at the ULA can be written as

$$\mathbf{u}(n)=u_{des}(n)\,\mathbf{s}(\phi_0)+\sum_{l=1}^{L-1}u_{known\_l}(n)\,\mathbf{s}(\phi_l) + \sum_{k=L}^{N}u_{unknown\_k}(n)\,\mathbf{s}(\phi_k) \quad (2)$$

where $u_{des}(n)$ denotes the desired signal, $u_{known\_l}(n)$ $(1\le l\le L-1)$ the *l*th interference known DOA, $u_{unknown\_k}(n)$ $(L\le k\le N)$ the *k*th interference unknown DOA.

In this model, we ignore the additive noise in the array, we concentrate on keeping the desired signal, cancelling the known DOA interference and decreasing the unknow DOA interference.

B. Scattering Arrival Angle

When scattering arrival angle happens, e.g, the antenna is placed on the wall [8], the arrival angle of signal is splitted into

$$\theta_{scat}=\bar{\theta}_{scat}+\delta_\theta \quad (3)$$

where $\bar{\theta}_{scat}$ is the nominal angle and δ_θ is the deviation. As θ_{scat} is a random variable, the deviation angle δ_θ remains a random variable.

For the uniform distribution, the pdf of δ_θ is:

$$p_{\delta_\theta}(\delta_\theta)=\begin{cases}\dfrac{1}{2\sqrt{3}\sigma_\theta}, & \delta_\theta\in(-\sqrt{3}\sigma_\theta,\,\sqrt{3}\sigma_\theta)\end{cases} \quad (4)$$

For the Gauss distribution, the pdf of δ_θ is:

$$p_{\delta_\theta}(\delta_\theta)=\begin{cases}\dfrac{1}{\sqrt{2\pi}\sigma_\theta}e^{-\frac{1}{2\sigma_\theta^2}\delta_\theta^2}, & \delta_\theta\in(-\frac{1}{2}\pi-\bar{\theta}_{0scat},\frac{1}{2}\pi-\bar{\theta}_{0scat})\end{cases} \quad (5)$$

where σ_θ is the standard deviation of the standard distribution.

III. GSC IMPLEMENTATION

A. GSC Algorithm

The output of GSC can be expressed as:

$$y(n)=\mathbf{w}^H\mathbf{u}(n) \quad (6)$$

where $\mathbf{w}=[w_0,\, w_1,...,w_{M-1}]^T$ is the weight vector,

$\mathbf{u}(n)=[u(n),\, u(n-1),...,\, u(n-M+1)]^T$ is the input vector.

Let $J=E\{|y(n)|^2\}$ denote a cost function if mean-squared error (MSE). The LCMV beamformer determines $\mathbf{w}$ by minimizing output power under appropriate linear weight constraints, which is given as

$$\min_{\mathbf{w}} J \text{ , subject to } \mathbf{C}^H\mathbf{w}=\mathbf{g} \quad (7)$$

where the $M\times L$ matrix $\mathbf{C}$ is the constraint matrix, $\mathbf{g}=[1,0,0,...,0]^T$ is a $L\times 1$ gain vector, with L is the number of constraints. With this gain vector $\mathbf{g}$, the beamfomer is constrained to preserve a signal of interest, at the same time, to suppress the interferences that known the arrival angle.

The GSC is an alternative formulation of the LCMV beamformer. It has been shown that the GSC can convert the constrained optimization into an unconstrained one. The structure of the GSC is illustrated in the Fig.1. As shown in the figure, the upper path includes the quiescent weight vector $\mathbf{w}_q$. The lower path includes the blocking matrix $\mathbf{C}_a$ and the interference cancelling filter $\mathbf{w}_a$.

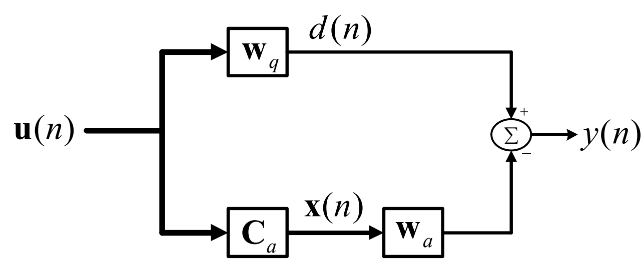

Figure 1. Block diagram of GSC

The $M\times 1$ quiescent weight vector $\mathbf{w}_q$, $\mathbf{w}_q=\mathbf{C}\left(\mathbf{C}^H\mathbf{C}\right)^{-1}\mathbf{g}$, sastifies the constraints. In contrast, the $(M-L)\times 1$ vector $\mathbf{w}_a$ is unaffected by the constraints.

We get: $\mathbf{w}=\mathbf{w}_q-\mathbf{C}_a\mathbf{w}_a$ (8)

The beamformer output:

$$y(n)=\mathbf{w}_q^H\mathbf{u}(n)-\mathbf{w}_a^H\mathbf{C}_a^H\mathbf{u}(n) \quad (9)$$

Define : $d(n)=\mathbf{w}_q^H\mathbf{u}(n)$ and $\mathbf{x}(n)=\mathbf{C}_a^H\mathbf{u}(n)$

We may rewrite in a form that resembles the standard Wiener filter: $y(n)=d(n)-\mathbf{w}_a^H\mathbf{x}(n)$ (10)

The $(M-L)\times(M-L)$ matrix $\mathbf{R}_\mathbf{x}$: $\mathbf{R}_\mathbf{x}=E\left[\mathbf{x}(n)\mathbf{x}^H(n)\right]$ (11)

The $(M-L)\times 1$ vector $\mathbf{p}_\mathbf{x}$: $\mathbf{p}_\mathbf{x}=E\left[\mathbf{x}(n)d^*(n)\right]$ (12)

Then, we can find the optimum $\mathbf{w}_a$: $\mathbf{w}_{a,opt}=\mathbf{R}_\mathbf{x}^{-1}\mathbf{p}_\mathbf{x}$ (13)

In practice, the covariance matrix $\mathbf{R_x}$ and the cross correlation vector $\mathbf{p_x}$ are estimated by :

$$\mathbf{R_x} = \frac{1}{N}\sum_{k=1}^{N}\mathbf{x}(n)\mathbf{x}^H(n) \tag{14}$$

$$\mathbf{p_x} = \frac{1}{N}\sum_{k=1}^{N}\mathbf{x}(n)d^*(n) \tag{15}$$

where N is the number samples.

B. Calculate Blocking Matrix

From the equation $\mathbf{C}^H\mathbf{C}_a = \mathbf{O}$, we'll find the blocking matrix $\mathbf{C}_a$. $\mathbf{C}_a$ is of dimension $M\times(M-L)$ and with the independent columns.

The constraint space is determined by the L independent column vectors of the constraint matrix $\mathbf{C}$. The projection matrix of this space is :

$$\mathbf{P_C} = \mathbf{C}\left(\mathbf{C}^H\mathbf{C}\right)^{-1}\mathbf{C}^H \tag{16}$$

Thus, the space that is orthogonal to the constraint space has the projection matrix given by:

$$\mathbf{P}_\mathbf{C}^{\perp} = \mathbf{I} - \mathbf{P_C} = \mathbf{I} - \mathbf{C}\left(\mathbf{C}^H\mathbf{C}\right)^{-1}\mathbf{C}^H \tag{17}$$

Clearly, $\mathbf{P}_\mathbf{C}^{\perp}$ sastifies the equation $\mathbf{C}^H\mathbf{C}_a = \mathbf{O}$, because of

$$\mathbf{C}^H.\mathbf{P}_\mathbf{C}^{\perp} = \mathbf{C}^H.(\mathbf{I} - \mathbf{C}\left(\mathbf{C}^H\mathbf{C}\right)^{-1}\mathbf{C}^H)$$

$$= \mathbf{C}^H.\mathbf{I} - \mathbf{C}^H.\mathbf{C}\left(\mathbf{C}^H\mathbf{C}\right)^{-1}\mathbf{C}^H = \mathbf{O}$$

Then, we use the Gram-Schmidt orthogonalization procedure to $\mathbf{P}_\mathbf{C}^{\perp}$ with M columns in order to determine the blocking matrix $\mathbf{C}_a$ with $(M-L)$ independent columns.

IV. Adjust GSC to Robust Against Scattering Arrival Angle

A. Robust Against Scattering Arrival Angle of Desired Signal

When the actual desired signal's DOA is scattered around the nominal angle $\bar{\theta}_{0scat}$:

$$\theta_{0scat} = \bar{\theta}_{0scat} + \delta_\theta$$

The desired signal become the interference for the optimum weight vector of the beamformer installed at this time. Therefore, both the output power of desired signal and SINR decreases.

To robust SINR against this mismatch condition, we need to do the GSC algorithm again with the new constraint. First, we need to replace the θ_0 distortionless constraint with the $\bar{\theta}_{0scat}$ distortionless constraint. Thus, the electrical angle of $\bar{\theta}_{0scat}$ is $\bar{\phi}_{0scat} = 2\pi\frac{d}{\lambda}\sin(\bar{\theta}_{0scat})$ and the corresponding steering vector is $s(\bar{\phi}_{0scat}) = \left[1 \quad e^{-j\bar{\phi}_{0scat}} \quad e^{-j2\bar{\phi}_{0scat}} \quad ... \quad e^{-j(M-1)\bar{\phi}_{0scat}}\right]^T$. Then, we adjust the constraint matrix $\mathbf{C}$ and keep the GSC algorithm as section III to have the optimum weight vector $\mathbf{w}_{o,scat}$ for the beamformer.

And we can find the optimum M antenna number of array to have the mainbeam width that is large enough to receive the signal corresponding to the scattering interval of the actual arrival angle. We can compare the scattering interval to mainbeam width by referring to Table I to choose the antenna number.

TABLE I. Mainbeam Width Correspongding To Antenna Number

M	8	9	10	11	12	13	14
BW	12.9^0	11.3^0	10.2^0	9.2^0	8.4^0	8.0^0	7.1^0
M	15	16	17	18	19	20	21
BW	6.5^0	6.3^0	5.9^0	5.6^0	5.3^0	5.0^0	4.8^0
M	22	23	24	25	26	27	28
BW	4.6^0	4.4^0	4.1^0	4.0^0	3.8^0	3.7^0	3.6^0
M	29	30					
BW	3.5^0	3.3^0					

a. M: Number of Antenna; BW: MainBeam Width

B. Robust Against Scattering Arrival Angle of Interference

When the actual interference's DOA is not the same as the presumed DOA, the beam pattern with the optimum weight vector installed has not reduced the power of interference as much as before. Although the sidelobes of beam pattern are very low to decrease the interference, SINR that interference's DOA is scattered is still much smaller than SINR in case of optimum.

To improve SINR in this case, we combine the GSC algorithm to find the optimum weight vector $\mathbf{w}_{opt}$ with the window functions to decrease the interference as much as possible. We make the Hadamard product of $\mathbf{w}_{opt}$ and $\mathbf{w}_{window}$ to have the new weight vector :

$$\mathbf{w}_{o,\text{int\_}scat}(n) = \mathbf{w}_{opt}(n)\times\mathbf{w}_{window}(n) \tag{18}$$

We use the Hamming window, Hanning window, Kaiser window and Chebyshev window [9] in this case and get the good results in beam pattern and SINR.

V. Numerical Examples

In examples 1 and 2 , we assume a uniform linear array (ULA) with $M = 16$ omnidirectional antennas spaced half a wavelength apart. The transmitted symbols are randomly generated from ± 1.

A. *Example 1: GSC algorithm*

We consider one desired source ($-60\,\text{dBm}$) coming from 0^0, four uncorrelated interfering sources (0 dBm each source, known arrival angle) coming from $-15^0, 20^0, -40^0, -65^0$ and one uncorrelated interfering source (0 dBm, unknown arrival angle) assumed coming from 60^0.

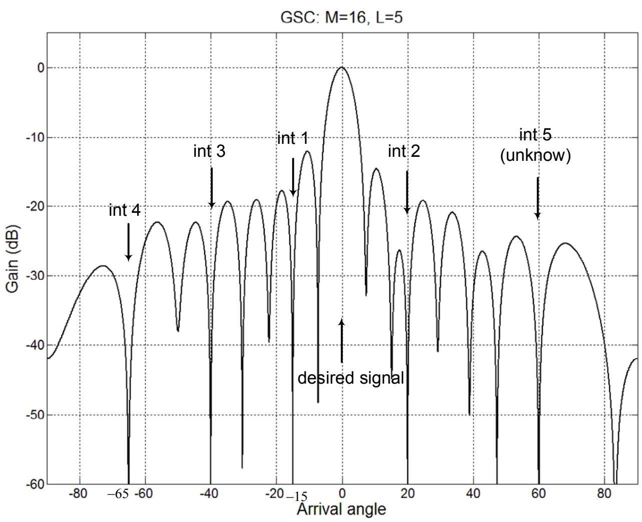

Figure 2. Beam pattern of GSC with 16 antennas, 5 constraints

TABLE II. SINR RESULT

SINR_element	SINR_optimum
-66.9897 dB	15.7188 dB

Figure 2 shows that the desired signal is preserved, the interferences 1, 2, 3, 4 are null at the known arrival angle and the unknown interference 5 is decreased so much. In table II, SINR also increases from -66.9897 dB to 15.7188 dB.

B. *Example 2: Scattering Arrival Angle of Desired Signal*

We consider one desired source ($-60\,\text{dBm}$) coming from 0^0, two uncorrelated interfering sources (0 dBm each source, known arrival angle) coming from 20^0, -15^0 and one uncorrelated interfering source (0 dBm, unknown arrival angle) assumed coming from 60^0.

Now, the desired signal comes with scattering arrival angle, The actual arrival angle is a random value θ_{0scat}, with $\theta_{0scat} = \bar{\theta}_{0scat} + \delta_\theta$, where $\bar{\theta}_{0scat}$ is -30^0 and the pdf of δ_θ is uniform in $[-3^0, 3^0]$.

We adjust the GSC algorithm as section IV.A, the results of beam pattern and SINR are shown in Figure 3 and Table III.

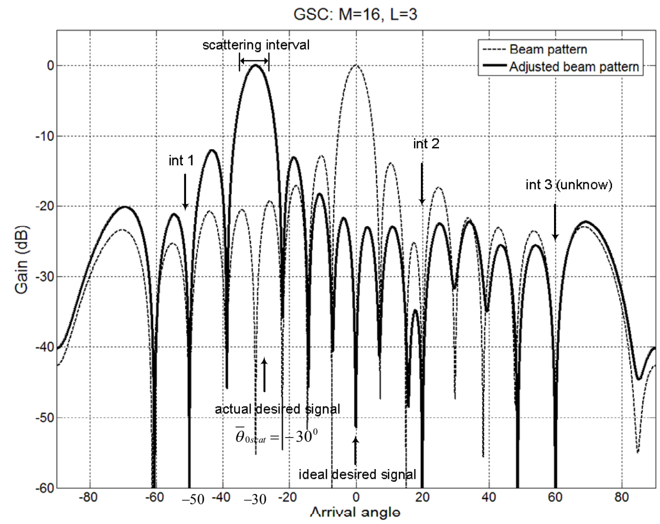

Figure 3. Beam pattern of adjusted GSC

TABLE III. SINR RESULT

SINR _element	**SINR_ optimum** (presumed desired signal)	**SINR_scattering** (actual desired signal and haven't adjusted GSC yet)	**SINR_adjusted** (actual desired signal and adjusted GSC)
-64.7712 dB	13.9383 dB	-7.4913 dB	11.5679 dB

Figure 3 shows that the beam pattern moves to the position that receives the scattered desired signal, the interferences 1, 2 are still in nulling. In table III, SINR also increases from -7.4913 dB to 11.5679 dB after GSC is adjusted.

C. *Example 3: Scattering Arrival Angle of Interference*

In case of using the window functions, we need the antenna number of beamformer to be large enough to have the effect of the window functions on the optimum weight vector. In this example, we simulate with $M = 30$ omnidirectional antennas

We consider one desired source ($-60\,\text{dBm}$) coming from 0^0, one uncorrelated interfering source (0 dBm, unknown arrival angle) assumed coming from 60^0.

Now, the interference comes with scattering arrival angle, The actual arrival angle is a random value θ_{1scat}, with $\theta_{1scat} = \bar{\theta}_{1scat} + \delta_\theta$, where $\bar{\theta}_{1scat}$ is 60^0 and the pdf of δ_θ is uniform distribution in $[-3^0, 3^0]$.

We adjust the GSC algorithm as section IV.B, the results of beam pattern and SINR are shown in Figure 4, 5 and Table IV.

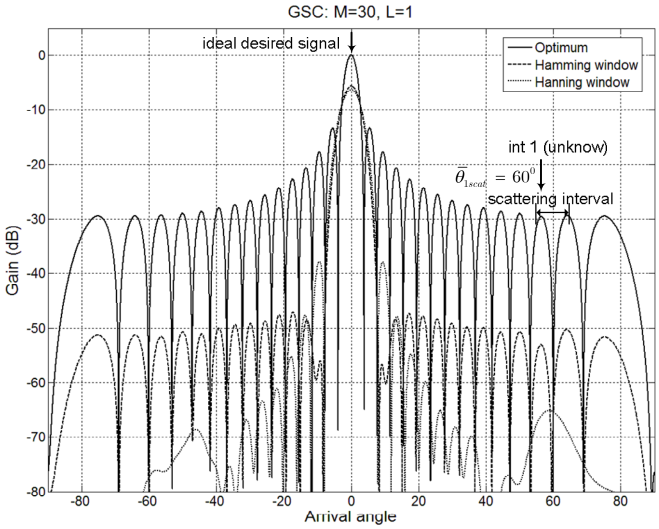

Figure 4. Beam pattern of GSC and Hamming, Hanning windows.

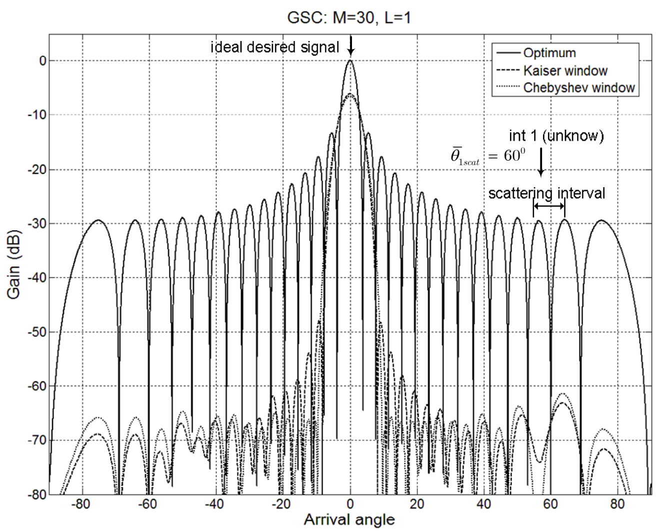

Figure 5. Beam pattern of GSC and Kaiser, Chebyshev windows.

TABLE IV. SINR RESULT

Antenna number	M = 30
SINR_element	-60 dB
SINR_optimum	32.3958 dB
SINR_scattering	-23.7096 dB
SINR_Hamming	-7.8536 dB
SINR_Hanning	-0.3132 dB
SINR_Kaiser	-0.2514 dB
SINR_Chebyshev	-2.9599 dB

Figures 4, 5 show that the sidelobes are decreased a lot with the window functions so the scattered interference is still reduced so much. In table IV, when using the windows function, SINR increases from -23.7096 dB to -7.8536 dB, -0.3132 dB, -0.2514 dB, -2.9599 dB according to Hamming, Hanning, Kaiser, Chebyshev windows, respectively.

VI. CONCLUSION

In this paper, we clearly show the general GSC algorithm to reduce interference with one desired signal, several known DOA interferences and several unknown DOA interferences for the optimum beamformer. And how to find the blocking matrix is presented. In the scattering desired signal's DOA situation, the GSC algorithm with the new desired steering vector needs to perform again to get the new optimum weight vectors. In the scattering interference's DOA situation, the window functions such as Hamming, Hanning, Kaiser and Chebyshev windows are combined with the optimum weight vector of the GSC algorithm to improve SINR. The performance are evaluated by beam pattern and SINR. The good result of our proposal has been demonstrated via three numerical examples with computer simulations.

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